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THE GLOBAL ATTRACTORS FOR A DAMPED GENERALIZED COUPLED NONLINEAR WAVE EQUATIONS

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Abstract The existence of global attractors for the periodic initial value problem of damped generalized coupled nonlinear wave equations is proved. We also get the estimates of the upper bounds of Hausdorff and fractal dimensions for the global attractors by means of a uniformly priori estimates for time.
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1. Introduction

In [1], the authors established the unique existence of the smooth solution for the following coupled nonlinear equations

$$u_t = u_{xxx} + buu_x + 2vv_x, \tag{1.1}$$

$$v_t = 2(uv)_x. \tag{1.2}$$

These were proposed to describe the interaction process of internal long waves. In [2], Ito M. proposed a recursion operator by which he inferred that the equations (1.1) and (1.2) possess infinitely many symmetries and constants of motion. In [3], P.F.He established the existence of a smooth solution to the system of coupled nonlinear KdV equation [4]

$$u_t = a(u_{xxx} + buu_x) + 2bvv_x, \tag{1.3}$$

$$v_t = -v_{xxx} - 3uv_x, \tag{1.4}$$

where a and b are constants.

We remark that M. E. Schonbek [5] dealt with a very similar system of coupled nonlinear equation [6]

$$u_t = u_{xxx} - uu_x - v_x, \tag{1.5}$$

$$v_t = -(uv)_x. \tag{1.6}$$

The global existence of a weak solution was established via the technique of parabolic regularization and Dunford's theorem on weakly sequentially compact L^1 sets.

In this paper, we consider the following periodic initial value problem of damped coupled nonlinear wave equations

$$u_t + f(u)_x - \alpha u_{xx} + \beta u_{xxx} + 2vv_x = G_1(u,v) + h_1(x), \tag{1.7}$$

$$v_t - \gamma v_{xx} + (2uv)_x + g(v)_x = G_2(u,v) + h_2(x), \tag{1.8}$$

$$u(x+D,t) = u(x-D,t), v(x+D,t) = v(x-D,t), x \in R, t \geq 0, \tag{1.9}$$

$$u(x,0) = u_0(x), v(x,0) = v_0(x), x \in R, \tag{1.10}$$

where $D > 0$, $\alpha > 0$, $\beta \neq 0$, $\gamma > 0$ are real numbers, and $\int_{-D}^{D} u(x,t)dx = 0, \int_{-D}^{D} v(x,t)dx = 0$. We establish the t-independent a priori estimates of the problem (1.7)-(1.10) and get the estimate of upper bounds of Hausdorff and fractal dimensions for the global attractor.

To simplify the notation in this paper, we shall denote by $\|\cdot\|$ the norm $\|\cdot\|_{L_2}$, by $\|\cdot\|_p$ the norm $\|\cdot\|_{L^p}$, by $\|\cdot\|_\infty$ the norm $\|\cdot\|_{L^\infty}$, by $\|\cdot\|_m$ the norm $\|\cdot\|_{H^m}$, $\Omega = (-D,D)$.

2. t-independent A Priori Estimates of Problem (1.7)-(1.10)

Lemma 1 *Suppose that*

(1) $G_i(0,0) = 0 \quad (i=1,2)$, $\quad (\xi,\eta)\begin{pmatrix} -G_{1u} & -G_{2u} \\ -G_{1v} & -G_{2v} \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} \geq b_0(|\xi|^2 + |\eta|^2)$, $(\xi,\eta) \in \mathrm{R}^2$, $b_0 > 0$ *is a constant,*

(2) $u_0 \in L^2(\Omega), v_0 \in L^2(\Omega), h_i(x) \in L^2(\Omega)(i=1,2), \Omega = (-D,D)$.

Then for the smooth solution of the problem (1.7)-(1.10), we have the following estimate

$$\|u(\cdot,t)\|^2 + \|v(\cdot,t)\|^2 \leq e^{-b_0 t}(\|u_0\|^2 + \|v_0\|^2) + \frac{1}{b_0^2}(1 - e^{-b_0 t})(\|h_1\|^2 + \|h_2\|^2). \tag{2.1}$$

Furthermore, we have

$$\overline{\lim_{t\to\infty}}(\|u(\cdot,t)\|^2 + \|v(\cdot,t)\|^2) \leq \frac{1}{b_0^2}(\|h_1\|^2 + \|h_2\|^2) = E_0, \tag{2.2}$$

$$\overline{\lim_{t\to\infty}}\frac{1}{t}\int_0^t [\alpha\|u_x(\cdot,\tau)\|^2 + \gamma\|v_x(\cdot,\tau)\|^2]d\tau \leq \frac{1}{b_0^2}(\|h_1\|^2 + \|h_2\|^2). \tag{2.3}$$

Proof Taking the inner product of (1.7) with u, (1.8) with v, then we have

$$(u, u_t + f(u)_x - \alpha u_{xx} + \beta u_{xxx} + 2vv_x) = (u, G_1(u,v) + h_1(x)), \tag{2.4}$$

$$(v, v_t - \gamma v_{xx} + (2uv)_x + g(v)_x) = (v, G_2(u,v) + h_2(x)), \tag{2.5}$$

where

$$
\begin{aligned}
&(u,w)=\int_{-D}^{D}u(x,t)w(x,t)dx,\quad (u,f(u)_x)=0,\\
&(u,-\alpha u_{xx})=\alpha\|u_x\|^2,\quad (v,-\gamma v_{xx})=\gamma\|v_x\|^2,\\
&(u,\beta u_{xxx})=0,\quad (v,g(v)_x)=0,\\
&(u,2vv_x)+(v,2(uv)_x)=2\int uvv_xdx-2\int uvv_xdx=0,\\
&(u,G_1(u,v))+(v,G_2(u,v))=(u,G_{1u}u+G_{1v}v)+(v,G_{2u}u+G_{2v}v)\\
&\qquad=(u,v)\begin{pmatrix}-G_{1u} & -G_{2u}\\ -G_{1v} & -G_{2v}\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}\le -b_0(\|u\|^2+\|v\|^2),\\
&(u,h_1(x))\le\frac{b_0}{2}\|u\|^2+\frac{1}{2b_0}\|h_1\|^2,\quad (v,h_2(x))\le\frac{b_0}{2}\|v\|^2+\frac{1}{2b_0}\|h_2\|^2.
\end{aligned}
$$

Summing up the equality (2.4) and (2.5), we get

$$\frac{1}{2}\frac{d}{dt}(\|u\|^2+\|v\|^2)+\alpha\|u_x\|^2+\gamma\|v_x\|^2+\frac{b_0}{2}(\|u\|^2+\|v\|^2)\le\frac{1}{2b_0}(\|h_1\|^2+\|h_2\|^2).\tag{2.6}$$

The inequality (2.6) implies (2.1)-(2.3).

Lemma 2 (Sobolev's Inequality [7]) *Suppose that* $u\in L_q(\Omega)$, *where* $1\le r$, $q<\infty$, $\Omega\in \mathrm{R}^n$. *Then there exists a constant* $C>0$, *such that*

$$\|D^ju\|_{L_p(\Omega)}\le C\|D^mu\|^a_{L_r(\Omega)}\|u\|^{1-a}_{L_q(\Omega)}$$

where $0\le j\le m$, $j/m\le a\le 1$, $1\le p\le\infty$ *and* $\frac{1}{p}=\frac{j}{n}+a(\frac{1}{r}-\frac{m}{n})+(1-a)\frac{1}{q}$.

Lemma 3 *Under the conditions of Lemma 1, we suppose that*

(1) $f(u)\in C^1$,$g(v)\in C^1$, $G_i(u,v)\in C^1(i=1,2)$ *and* $|f(u)|\le A|u|^{5-\delta}$, $|g(v)|\le B|v|^{6-\delta}$, $A>0$, $B>0$, $\delta\ge 0$, $|G_i|\le C_i(|u|^5+|v|^5)$, $C_i>0$;

(2) $u_{0x}(x)\in L^2(\Omega), v_{0x}(x)\in L^2(\Omega)$.

Then for the smooth solution of the problem (1.7)-(1.10), we have the estimate

$$
\begin{aligned}
\|u_x\|^2+\|v_x\|^2\le & 2e^{-2b_0t}(\|u_{0x}\|^2+\|v_{0x}\|^2-\frac{2}{\beta}\int F(u_0(x))dx)+2e^{-2b_0t}\int_0^tC_1e^{2b_0\tau}d\tau\\
&+\frac{1}{b_0}(1-e^{-2b_0t})(\frac{3}{\alpha}\|h_1\|^2+\frac{4}{\gamma}\|h_2\|^2)+C_2,
\end{aligned}\tag{2.7}
$$

where $F(u)=\int_0^uf(s)ds$ *,the functions* $C_1(\cdot)$ *and* $C_2(\cdot)$ *depend on* $\|u\|$,$\|v\|$. *Furthermore, we have*

$$\overline{\lim_{t\to\infty}}(\|u_x(\cdot,t)\|^2+\|v_x(\cdot,t)\|^2)\le\frac{1}{b_0}\max_{t\ge0}C_1+\frac{1}{b_0}(\frac{3}{\alpha}\|h_1\|^2+\frac{4}{\gamma}\|h_2\|^2)+\max_{t\ge0}C_2\equiv E_1,\tag{2.8}$$

$$\overline{\lim_{t\to\infty}}\frac{1}{t}\int_0^t[\alpha\|u_{xx}(\cdot,\tau)\|^2+\gamma\|v_{xx}(\cdot,\tau)\|^2]d\tau\leq\max_{t\geq 0}(b_0C_2+C_1)\frac{1}{b_0^2}+\frac{3}{\alpha}\|h_1\|^2+\frac{4}{\gamma}\|h_2\|^2. \tag{2.9}$$

Proof Taking the inner product of (1.7) with u_{xx} it follows that

$$(u_{xx},u_t+f(u)_x-\alpha u_{xx}+\beta u_{xxx}+2vv_x)=(u_{xx},G_1(u,v)+h_1(x)), \tag{2.10}$$

where

$$\begin{aligned}(u_{xx},u_t)=&-\frac{1}{2}\frac{d}{dt}\|u_x\|^2\\(u_{xx},f(u)_x)=&-(u_{xxx},f(u))\\=&\frac{1}{\beta}(u_t+f(u)_x-\alpha u_{xx}+2vv_x-G_1-h_1,f(u))\\=&\frac{1}{\beta}\frac{d}{dt}\int F(u)dx-\frac{\alpha}{\beta}(u_{xx},f(u))+\frac{2}{\beta}(vv_x,f(u))-\frac{1}{\beta}(G_1+h_1,f(u)).\end{aligned}$$

By using the Sobolev interpolation inequality, we get

$$\begin{aligned}|(u_{xx},f(u))|\leq&\|u_{xx}\|\cdot\|f(u)\|\leq A\|u_{xx}\|\cdot\|u\|_{10-2\delta}^{5-\delta}\leq\frac{\beta}{12}\|u_{xx}\|^2+C(\|u\|),\\\frac{2}{\beta}|(vv_x,f(u))|\leq&\frac{2}{|\beta|}\|v\|_4\|v_x\|\|f(u)\|_2\leq\frac{\alpha}{12}\|u_{xx}\|^2+\frac{\gamma}{8}\|v_{xx}\|^2+C(\|u\|,\|v\|),\end{aligned}$$

$$\begin{aligned}|\frac{1}{\beta}(G,f(u))|\leq&\frac{4C_1}{|\beta|}(\|u\|_{10-\delta}^{10-\delta}+\|v\|_{10-2\delta}^{5-\delta}+\|v\|_{10}^{5})\\\leq&\frac{\alpha}{12}\|u_{xx}\|^2+\frac{\gamma}{8}\|v_{xx}\|^2+C(\|u\|,\|v\|),\\|(u_{xx},h_1)|\leq&\|u_{xx}\|\cdot\|h_1\|\leq\frac{\alpha}{12}\|u_{xx}\|^2+\frac{3}{\alpha}\|h_1\|^2,\\(u_{xx},-\alpha u_{xx})=&-\alpha\|u_{xx}\|^2,\quad(u_{xx},\beta u_{xxx})=0.\end{aligned}$$

From (2.10), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}[\|u_x\|^2-\frac{2}{\beta}\int F(u)dx]+\frac{7\alpha}{12}\|u_{xx}\|^2-2(u_{xx},vv_x)\\&\leq-(u_{xx},G_1(u,v))+\frac{\gamma}{4}\|u_{xx}\|^2+\frac{3}{\alpha}\|h_1\|^2+C.\end{aligned} \tag{2.11}$$

Taking the inner product of (1.8) with v_{xx}, it follows that

$$(v_{xx},v_t+g(v)_x-\gamma v_{xx}+2(vv)_x)=(v_{xx},G_2(u,v)+h_2(x)), \tag{2.12}$$

where

$$(v_{xx},v_t)=-\frac{1}{2}\frac{d}{dt}\|v_x\|^2\quad(v_{xx},-\gamma v_{xx})=-\gamma\|v_{xx}\|^2,$$

$$|(v_{xx},g(v)_x)|\leq C\|v_{xx}\|\cdot\|g(v)_x\|\leq\frac{\gamma}{16}\|v_{xx}\|^2+C,$$

$$|(v_{xx}, h_2)| \le \|v_{xx}\| \cdot \|h_2\| \le \frac{\gamma}{16}\|v_{xx}\|^2 + \frac{4}{\gamma}\|h_2\|^2,$$

$$|2(u_{xx}, vv_x) + 2(v_{xx}, (uv)_x)| = 3|\int u_x v_x^2 dx| \le 3\|u_x\|^2\|v_x\|_4^2 \le \frac{\alpha}{24}\|u_{xx}\|^2 + \frac{\gamma}{16}\|v_{xx}\|^2 + C,$$

$$\begin{aligned}(u_{xx}, G_1) + (v_{xx}, G_2) &= -(u_x, G_{1x}) - (v_x, G_{2x}),\\ &= -(u_x, G_{1u}u_x + G_{1v}v_x) - (v_x, G_{2u}u_x + G_{2v}v_x)\\ &= -(u_x, v_x)\begin{pmatrix} G_{1u} & G_{2u} \\ G_{1v} & G_{2v}\end{pmatrix}\begin{pmatrix} u_x \\ v_x\end{pmatrix} \le -b_0(\|u_x\|^2 + \|v_x\|^2).\end{aligned}$$

Summing up the inequality (2.11) with (2.12), then we have

$$\frac{d}{dt}\phi(t) + 2b_0\phi(t) + \alpha\|u_{xx}\|^2 + \gamma\|v_{xx}\|^2 \le \frac{3}{\alpha}\|h_1\|^2 + \frac{4}{\gamma}\|h_2\|^2 + C, \tag{2.13}$$

where $\phi(t) = \|u_x\|^2 + \|v_x\|^2 - \frac{2}{\beta}\int F(u)dx$

Integrating the inequality (2.13), we get

$$\phi(t) \le e^{-2b_0}\phi(0) + e^{-2b_0 t}\int_0^t Ce^{2b_0 s}ds + \frac{1}{2b_0}(1 - e^{-2b_0 t})(\frac{3}{\alpha}\|h_1\|^2 + \frac{4}{\gamma}\|h_2\|^2).$$

Noting

$$|\frac{2}{\beta}\int_\Omega F(u)dx| \le A\|u\|_{6-\delta}^{6-\delta} \le \frac{1}{2}\|u_{xx}\|^2 + C,$$

we have

$$\|u_x\|^2+\|v_x\|^2 \le 2e^{-2b_0 t}\phi(0)+2e^{-2b_0 t}\int_0^t C_1 e^{2b_0 s}ds+\frac{1}{b_0}(1-e^{-2b_0 t})(\frac{3}{\alpha}\|h_1\|^2+\frac{4}{\gamma}\|h_2\|^2+C_2), \tag{2.14}$$

where the functions C_1 and C_2 depend on $\|u\|$ and $\|v\|$. From (2.14) we can get (2.8), (2.9).

Lemma 4 *Under the conditions of Lemma 3, we assume that*

(1) $f(u) \in C^2, g(v) \in C^2, G_i \in C^2 (i = 1, 2)$

(2) $u_0(x) \in H^2(\Omega), v_0(x) \in H^2(\Omega), h_i(x) \in H^1(\Omega)(i = 1, 2)$.

Then for the smooth solution of problem (1.7)-(1.10) we have

$$\begin{aligned}\|u_{xx}\|^2 + \|v_{xx}\|^2 \le{}& e^{-2b_0 t}(\|u_{0xx}\|^2 + \|v_{0xx}\|^2) + 2e^{-2b_0 t}\int_0^t C_3 e^{2b_0 s}ds\\ &+\frac{1}{2b_0}(1 - e^{-2b_0 t})(\frac{5}{\alpha}\|h_{1x}\|^2 + \frac{2}{\gamma}\|h_{2x}\|^2)\end{aligned} \tag{2.15}$$

where the function C_3 depends on $\|u\|_{H^1}$ and $\|v\|_{H^1}$. Furthermore, we have

$$\overline{\lim_{t\to\infty}}(\|u_{xx}\|^2 + \|v_{xx}\|^2) \le \frac{1}{2b_0}\max_{t\ge 0} C_3 + \frac{1}{2b_0}(\frac{5}{\alpha}\|h_{1x}\|^2 + \frac{2}{\gamma}\|h_{2x}\|^2) = E_2, \tag{2.16}$$

$$\overline{\lim_{t\to\infty}}\frac{1}{t}\int_0^t[\alpha\|u_{xxx}\|^2+\gamma\|v_{xxx}\|^2]ds\le\max_{t\ge0}C_3+[\frac{5}{\alpha}\|h_{1x}\|^2+\frac{2}{\gamma}\|h_{2x}\|^2]. \tag{2.17}$$

Proof Taking the inner product of(1.7) with u_{xxxx}, we have

$$(u_{xxxx},u_t+f(u)_x-\alpha u_{xx}+\beta u_{xxx}+2vv_x)=(u_{xxxx},G_1+h_1), \tag{2.18}$$

where

$$\begin{aligned}|(u_{xxxx},f(u)_x)|=&|(u_{xxx},f''(u)u^2(x)+f'(u)u_{xx})|\\ \le&\|f''(u)\|_\infty\|u_{xxx}\|\|u_x\|_4^2+\|f'(u)\|_\infty\|u_{xxx}\|\|u_{xx}\|\\ \le&\frac{\alpha}{10}\|u_{xxx}\|^2+C, \qquad\qquad (2.19)\\ |(u_{xxxx},-\alpha u_{xx})|=&\alpha\|u_{xxx}\|^2,\quad (u_{xxxx},\beta u_{xxx})=0,\\ |(u_{xxxx},2vv_x)|=&|(u_{xxx},2(v_x^2+vv_x))|\le\frac{\gamma}{8}\|v_{xxx}\|^2+\frac{\alpha}{10}\|u_{xxx}\|^2+C,\\ |(u_{xxxx},h_1)|\le&\|u_{xxx}\|\|h_{1x}\|\le\frac{5}{\alpha}\|h_{1x}\|^2+\frac{\alpha}{10}\|u_{xxx}\|^2.\end{aligned}$$

Taking the inner product of (1.8) with v_{xxxx}, we have

$$(v_{xxxx},v_t-\gamma v_{xx}+2(uv)_x+g(v)_x)=(v_{xxxx},G_2+h_2). \tag{2.20}$$

Note

$$|(v_{xxxx},-\gamma v_{xx})|=\gamma\|v_{xxx}\|^2,$$

$$\begin{aligned}|(v_{xxxx},2(uv)_x)|=&|(v_{xxx},2(u_xv+uv_x))|=|(v_{xxx},2(u_{xx}v+2u_xv_x+uv_{xx}))|\\ \le&\frac{\gamma}{16}\|v_{xxx}\|^2+\frac{\alpha}{10}\|u_{xxx}\|^2+C,\end{aligned}$$

$$\begin{aligned}|(v_{xxxx},g(v)_x)|=&|(v_{xxx},g''(v)v^2(x)+g'(v)v_{xx})|\\ \le&\|g''(v)\|_\infty\|v_{xxx}\|\|v_x\|_4^2+\|g'(v)\|_\infty\|v_{xxx}\|\|v_{xx}\|\\ \le&\frac{\gamma}{16}\|v_{xxx}\|^2+C,\\ |(v_{xxxx},h_2)|\le&\|v_{xxx}\|\|h_{2x}\|\le\frac{2}{\gamma}\|h_{2x}\|^2+\frac{\gamma}{8}\|u_{xxx}\|^2,\end{aligned}$$

$$\begin{aligned}(u_{xxxx},G_1)+(v_{xxxx},G_2)=&(u_{xx},G_{1u}u_{xx}+G_{1v}v_{xx}+G_{1uu}u_x^2+G_{1vv}v_x^2+2G_{1uv}u_xv_x)\\ &+(v_{xx},G_{2u}u_{xx}+G_{2v}v_{xx}+G_{2uu}u_x^2+G_{2vv}v_x^2+2G_{2uv}u_xv_x)\\ \le&-b_0(\|u_{xx}\|^2+\|v_{xx}\|^2)+\frac{\gamma}{8}\|v_{xxx}\|^2+\frac{\alpha}{10}\|u_{xxx}\|^2+C,\end{aligned}$$

where the funtion C depends on $\|u\|_{H^1}$ and $\|v\|_{H^1}$. From (2.18) and (2.20),we get

$$\begin{aligned}\frac{d}{dt}(\|u_{xx}\|^2+\|v_{xx}\|^2)+&\alpha\|u_{xxx}\|^2+\gamma\|v_{xxx}\|^2+2b_0(\|u_{xx}\|^2+\|v_{xx}\|^2)\\ &\le\frac{5}{\alpha}\|h_{1x}\|^2+\frac{2}{\gamma}\|h_{2x}\|^2+C.\end{aligned} \tag{2.21}$$

Thus we have (2.15). From (2.15), we can get (2.16),(2.17).

Lemma 5 *Under the conditions of Lemma 4, we suppose that*

(1) $f(u) \in C^3$,$g(v) \in C^3$, $G_i(u,v) \in C^3 (i = 1, 2)$,

(2) $u_{0x} \in H^3(\Omega), v_{0x} \in H^3(\Omega), h_i(x) \in H^2(\Omega)(i = 1, 2)$.

Then for the smooth solution of the problem (1.7)-(1.10), we have the following estimates

$$\|u_{xxx}\| + \|v_{xxx}\| \le \frac{E_3}{t}, \quad t > 0, \tag{2.22}$$

where E_3 *depends on* $\|u_0\|_{H^2}$,$\|v_0\|_{H^2}$,$\|h_i\|_{H^2}$,$(i = 1, 2)$ *and* t.

Proof Taking the inner product of (1.7) with $t^2 u_{x^6}$, (1.8) with $t^2 v_{x^6}$,it follows that

$$(t^2 u_{x^6}, u_t + f(u)_x - \alpha u_{xx} + \beta u_{xxx} + 2vv_x) = (t^2 u_{x^6}, G_1 + h_1) \tag{2.23}$$

$$(t^2 v_{x^6}, v_t + g(v)_x - \gamma v_{xx} + 2(uv)_x) = (t^2 v_{x^6}, G_2 + h_2). \tag{2.24}$$

Since

$$(t^2 u_{x^6}, u_t) = \frac{1}{2}\frac{d}{dt}\|tu_{xxx}\|^2 + \|\sqrt{t}u_{xxx}\|^2,$$

$$(t^2 v_{x^6}, v_t) = \frac{1}{2}\frac{d}{dt}\|tv_{xxx}\|^2 + \|\sqrt{t}v_{xxx}\|^2,$$

by using the inequality (see [7])

$$\|f(u)\|_k \le C(\|u\|_\infty + \|u\|_\infty^{k-1} + 1) \max_{1\le p\le k} |D^p f(u)|\|u\|_k,$$

where the constant C is independent of f and u, we have

$$|(t^2 u_{x^6}, f(u)_x)| \le Ct^2\|u_{xxx}\|\|u_{xxxx}\| \le \frac{\alpha}{8}\|tu_{xxxx}\|^2 + C(\|tu_{xxx}\|^2 + 1),$$

$$|(t^2 v_{x^6}, g(v)_x)| \le Ct^2\|v_{xxx}\|\|v_{xxxx}\| \le \frac{\gamma}{8}\|tv_{xxxx}\|^2 + C(\|tv_{xxx}\|^2 + 1),$$

$$|(t^2 u_{x^6}, G_1 + h_1)| \le \frac{\alpha}{8}\|tu_{xxxx}\|^2 + C,$$

$$|(t^2 v_{x^6}, G_2 + h_2)| \le \frac{\gamma}{8}\|tv_{xxxx}\|^2 + C,$$

$$\begin{aligned}
|(t^2 u_{x^6}, 2vv_x)| =& |(t^2 u_{xxxx}, 6v_x v_{xx} + 2vv_{xxx})| \le \frac{\alpha}{8}\|tu_{xxxx}\|^2 + \frac{\gamma}{8}\|tv_{xxxx}\|^2 + C,\\
|(t^2 v_{x^6}, 2(uv)_x)| =& |(t^2 v_{xxxx}, 2(u_{xxx}v + 3u_{xx}v_x + 3uv_{xx} + uv_{xxx}))|\\
\le& \frac{\alpha}{8}\|tu_{xxxx}\|^2 + \frac{\gamma}{8}\|tv_{xxxx}\|^2 + C,
\end{aligned}$$

where the function C depends on $\|u\|_{H^2}$,$\|v\|_{H^2}$ and t, we get

$$\frac{d}{dt}(\|tu_{xxx}\|^2 + \|tv_{xxx}\|^2) + \alpha\|tu_{xxxx}\|^2 + \gamma\|tv_{xxxx}\|^2 \le C(\|tu_{xxx}\|^2 + \|tv_{xxx}\|^2 + 1). \tag{2.25}$$

Then from (2.25), we can get (2.22).

3. Existence of Global Smooth Solution and Global Attractor

We use the Galerkin method to establish the existence of the approximate solution for the problem (1.7)-(1.10).

Let $\omega_j(x)$ $(j=1,2,\cdots)$ be the normalized eigenfunctions of the equation $\Delta U+\lambda u=0$ with the periodic initial value (1.9),(1.10) and λ_j $(j=1,2,\cdots)$ are the corresponding eigenvalues. Then $\{\omega_j(x)\}$ forms a normalized orthogonal basis in L^2.

Denote the approximate solution of the problem (1.7)-(1.10) by $u_N(x,t)$, $v_N(x,t)$ in the form

$$u_N(x,t)=\sum_{j=1}^{N}\alpha_{jN}(t)\omega_j(x), \quad v_N(x,t)=\sum_{j=1}^{N}\beta_{jN}(t)\omega_j(x), \tag{3.1}$$

where $\alpha_{jN}(t)$, $\beta_{jN}(t)$ $(j=1,2,\cdots,N_j;N=1,2,\cdots)$ $(t\in R^+)$ are the functions satisfying the following system of ordinary equations of first order

$$(u_{Nt}+f(u_N)_x-\alpha u_{Nxx}+\beta u_{Nxxx}+2v_Nv_{Nx}-G_1(u_N,v_N)-h_1(x),\omega_j(x))=0, \tag{3.2}$$

$$(v_{Nt}+g(v_N)_x-\gamma v_{Nxx}+2(u_Nv_N)_x-G_2(u_N,v_N)-h_2(x),\omega_j(x))=0 \tag{3.3}$$

and the initial condition

$$(u_N(x,0),\omega_j(x))=(u_0(x),\omega_j(x)),$$
$$(v_N(x,0),\omega_j(x))=(v_0(x),\omega_j(x)), \tag{3.4}$$

obviously there holds

$$(u_{Nt}(x,0),\omega_j(x))=(u_N(x,0),\omega_j(x))=\alpha'_{jN}(0),$$
$$(v_{Nt}(x,0),\omega_j(x))=(v_N(x,0),\omega_j(x))=\beta'_{jN}(0).$$

By the similar a priori estimate we know that there exists a global solution for the initial value problem of the nonlinear ordinary differential system (3.2)-(3.4) on $[0,T]$.

Theorem 3.1 *Suppose that the following conditions are satisfied,*

(1) $G_i(0,0)=0(i=1,2)$, $(\xi,\eta)\begin{pmatrix}-G_{1u} & -G_{2u}\\ -G_{1v} & -G_{2v}\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}\geq b_0(|\xi|^2+|\eta|^2)$, $(\xi,\eta)\in \mathrm{R}^2$, b_0 *is a constant,*

(2) $f(u)\in C^k$,$g(v)\in C^k$, $G_i(u,v)\in C^k(i=1,2)$, $|f(u)|\leq A|u|^{5-\delta}$, $|g(v)|\leq B|v|^{6-\delta}$, $|G_i|\leq C_0(|u|^5+|v|^5)$, $h_i(x)\in H^{k-1}$ $(i=1,2)$, $A>0$, $B>0$, $C_0>0$, $k\geq 3$,

(3) $u_0(x)\in H^k(\Omega),v_0(x)\in H^k(\Omega)$, $k\geq 3$.

Then there exists a unique global smooth solution $u(x,t)$, $v(x,t)$ *for the problem (1.7)-(1.10) and* $u(x,t)\in L^\infty(0,T;H^k)$,$v(x,t)\in L^\infty(0,T;H^k)$.

Proof Similar to the proof of Lemma 1-Lemma 5, we have

$$\sup_{0\leq t\leq T}(\|u_N(x,t)\|_{H^k}+\|v_N(x,t)\|_{H^k})\leq C_k,$$

where constants C_k are independent of N. Let $U_N(x,t)=\begin{pmatrix} u_N(x,t)\\ v_N(x,t)\end{pmatrix}$. Thus we get the existence of the approximate solution for the problem (3.2)-(3.4). Furthermore, from the approximate solution sequence $v_N(x,t)$ we can choose subsequence $v_{N_i}(x,t)$ and function $U(x,t)\in L^\infty(0,T,H^k)$,

$$U_{N_i}(x,t)\to U(x,t)\quad \text{weakly star in } \mathrm{L^\infty(0,T;H^k)}\ \ (\mathrm{k}\geq 3).$$

From

$$(u_{Nt},u_{Nt}+f(u_N)_x-\alpha u_{Nxx}+\beta u_{Nxxx}+2v_Nv_{Nx}-G_1(u_N,v_N)-h_1(x))=0, \quad (3.5)$$

$$(v_{Nt},v_{Nt}+g(v_N)_x-\gamma v_{Nxx}+2(u_Nv_N)_x-G_2(u_N,v_N)-h_2(x))=0, \quad (3.6)$$

we can get

$$\|u_{Nt}\|+\|v_{Nt}\|\leq C'_k,$$

where constants C'_k are independent of N. Thus we have

$$U_{N_i}\to U_t\quad \text{weakly star in } \mathrm{L^\infty(0,T;L_2(\Omega)),N_i\to\infty}.$$

Thus the functions $u(x,t),v(x,t)$ satisfy the problem (1.7)-(1.10) a.e.. There exists a global smooth solution for the problem (1.7)-(1.10), it is easy to prove that the global solution is unique.

In order to prove the existence of global attractor of the problem (1.7)-(1.10), we need the following Babin-Vishik's result (see [8]).

Theorem 3.2 *Let E be a Banach space. Let $\{S_t,t\geq 0\}$ be a set of semi-group operators, i.e.,S_t: $E\to E$ satisfy*

$$S_tS_\tau=S_{t+\tau},\quad S_0=I,$$

where I is the identity operator. We also assume that

(1) *Operator S_t is bounded, i.e., for each $R>0$, there exists a constant $C(R)$ such that $\|u\|_E\leq R$ implies*

$$\|S_tu\|_E\leq C(R)\ \ \text{for}\ \ \mathrm{t}\in[0,\infty),$$

(2) *There is a bounded absorbing set $B_0\subset E$,i.e., for any bounded set $B\subset E$, there exists a constant T, such that*

$$S_tB\subset B_0,\quad \text{for}\ \ \mathrm{t}\geq \mathrm{T},$$

(3) *S_t is a completely continuous operator for $t>0$. Then the operator Semi-group S_t has a compact global attractor.*

Theorem 3.3 *Suppose that the problem* (1.7)-(1.10) *has a global smooth solution and assume that*

(1) $f(u) \in C^3$,$g(v) \in C^3$, $G_i(u,v) \in C^2(i=1,2)$, $|f(u)| \le A|u|^{5-\delta}$, $|g(v)| \le B|v|^{6-\delta}$, $|G_i| \le C_0(|u|^5+|v|^5)$, $(i=1,2)$, $A>0$, $B>0$, $\delta>0$

(2) $G_i(0,0)=0(i=1,2)$, $(\xi,\eta)\begin{pmatrix} -G_{1u} & -G_{2u} \\ -G_{1v} & -G_{2v} \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} \ge b_0(|\xi|^2+|\eta|^2)$, $(\xi,\eta)\in \mathrm{R}^2$, *$b_0$ is a constant*

(3) $u_0(x)\in H^2(\Omega), v_0(x)\in H^2(\Omega)$ $h_i(x)\in H^1(\Omega)$ $(i=1,2)$.

Then there exists a global attractor A of the periodic initial value problem (1.7)-(1.10),i.e., there is a set A, such that

(a) $S_tA=A$, *for* $t\in R^+$

(b) $\lim\limits_{t\to\infty}\mathrm{dist(S_tB,A)}=0$, *for any bounded set $B\subset H^2(\Omega)$, where*

$$\mathrm{dist(S_tB,A)}=\sup_{\mathrm{x\in B}}\inf_{\mathrm{y\in A}}\|\mathrm{x-y}\|_{\mathrm{E}}$$

and S_t is a semi-group operator generator generated by the problem (1.7)-(1.10).

Proof On account of the result of Theorem 3.2, we shall prove this theorem by checking the conditions (1)-(3) in Theorem 3.2

Under the assumptions of the theorem, we know that there exists an operator semi-group generated by the problem (1.7)-(1.10). Thus we set the Banach space $E=H^2(\Omega)$, and $S_t: H^2(\Omega)\to H^2(\Omega)$. By using the results of Lemma 1-5, and assuming that $B\subset H^2(\Omega)$ belongs to the ball $\{\|u\|_{H^2}+\|v\|_{H^2}\le R\}$, we have

$$\begin{aligned}\|S_t(u_0,v_0)\|_E^2=&\|u\|_{H^2}^2+\|v\|_{H^2}^2\le\|u_0\|_{H^2}^2+\|v_0\|_{H^2}^2+C_1(\|h_1\|_{H^1}^2+\|h_2\|_{H^1}^2)\\ \le&R^2+C_2, t\ge0,\quad u_0\in B,\quad v_0\in B,\end{aligned}$$

where C_1,C_2 are absolute constants. This means that $\{S_t\}$ is uniformly bounded in H^2. Furthermore, from the results of the above Lemmas we see that

$$\|S_t(u_0,v_0)\|_E^2=\|u\|_{H^2}^2+\|v\|_{H^2}^2\le 2(E_0+E_1+E_2), \tag{3.7}$$

$\forall\ \ t\ge t_0=T_0(R,\|u_0\|_{H^2},\|v_0\|_{H^2},\|h_1(x)\|_{H^1},\|h_2(x)\|_{H^1})$, Hence

$$\overline{A}=\{u\in H^2(\Omega),v\in H^2(\Omega),\|u\|_{H^2}+\|v\|_{H^2}\le 2(E_0+E_1+E_2)\}$$

is a bounded absorbing set of the operator semi-group S_t, and from Lemma 5 we see that

$$\|u_{xxx}\|+\|v_{xxx}\|\le\frac{E_3(R,t)}{t},\quad t>0$$

for $\|u_0\|_{H^2}\le R$, $\|v_0\|_{H^2}\le R$. By using the compact imbedding: $H^3(\Omega)\hookrightarrow H^2(\Omega)$, we thus know that the operator semi-group $S_t: H^2\to H^2$ for $t>0$ is completely continuous. The proof of the theorem is now completed.

Remark Just as the remarks pointed in [9], the attractor A obtained in Theorem 3.3 is the ω-limit set of the absorbing set $\overline{A}$, i. e.,

$$A=\omega(\overline{A})=\cap_{tau\ge0}\overline{\cup_{t\ge0}S_t\overline{A}}\ . \tag{3.8}$$

4. Upper Bound of Dimensions of Global Attractor

In order to establish the upper bounds of Hausdorff and fractal for the global attractor of the periodic initial value problem (1.7)-(1.10). We need the following linear variation corresponding to the problem (1.7)-(1.10):

$$\nu_t + L(u,v)\nu = 0, \tag{4.1}$$

$$\nu(0) = \nu_0, \tag{4.2}$$

where

$$\nu = \begin{pmatrix} \eta(x) \\ \xi(x) \end{pmatrix}, \quad \nu_0 = \begin{pmatrix} \eta(x) \\ \xi(x) \end{pmatrix},$$

$$L(u,v)\nu = \begin{pmatrix} -\alpha\eta_{xx} + \beta\eta_{xxx} + (f'(u)\eta)_x + 2(v\xi)_x - G_{1u}\xi - G_{1v}\xi \\ -\gamma\xi_{xx} + (g'(v)\xi)_x + 2(u\xi + v\eta)_x - G_{2u}\xi - G_{2v}\xi \end{pmatrix}.$$

Since the solution of the problem (1.7)-(1.10) is sufficiently smooth, we can easily prove that the linear problem (4.1)-(4.2) has a global smooth solution as long as the initial data are mildly smooth, i.e., there is a solution operator G_t such that $\nu(t) = G_t\nu_0$. It can be verified that the semi-group operator $S_t u_0, S_t v_0$ can be differentiated in $L_2(\Omega)$, namely, the Frechet derivative $S_t'U$ exists, and $G_t\nu_0 = S_tU$, $U = \begin{pmatrix} u \\ v \end{pmatrix}$, $U_0 = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$. In fact, we set

$$\omega(t) = S_\tau(U_0 + \nu_0) - S_t(U_0) - G_t(U_0)\nu_0 = U_1(t) - U(t) - \nu(t).$$

Thus we have

$$\partial_t\omega(t) = L_1(U_1) - L_1(U) + L(U)\nu(t) + L_1(U + \nu + \omega) - L_1(U) + L(U)\nu, \tag{4.3}$$

$$\omega(0) = 0, \tag{4.4}$$

where $U_t = L_1(U)$ is the operator form of the equation (1.7)-(1.8).Therefore,(4.3) can be rewritten as the form

$$\partial_t\omega(t) + L(U)\omega = \Lambda_0(U,\nu,\omega), \tag{4.5}$$

where

$$\Lambda_0(U,\nu,\omega) = L_1(U + \nu + \omega) - L_1(U) + L(U)(\nu + \omega). \tag{4.6}$$

By the theory of linear partial differential equations, we have the L_2-estimate

$$\|\omega(t)\| \le C\|\nu_0\|^2. \tag{4.7}$$

This implies that the semi-group operator S_t is differentiable in $L_2(\Omega)$.

Denote by $\nu_1(t),\cdots,\nu_J(t)$ the solutions of the linear equation (4.1) corresponding respectively to the initial data $\nu_1(0)=\xi_1,\cdots,\nu_J(0)=\xi_J$, where $\xi=(\xi_1,\xi_2,\cdots,\xi_J)\in L_2(\Omega)$. By simple computations (see [9]),we can deduce that

$$\frac{d}{dt}\|\nu_1(t)\wedge\cdots\wedge\nu_J(t)\|^2+2Tr\left(L(U)Q_J\right)\|\nu_1(t)\wedge\cdots\wedge\nu_J(t)\|^2=0, \tag{4.8}$$

where $L(U)=L(S_tU_0)$ is a linear map, $\nu\to L(U)\nu$; " $\wedge$" denotes the exterior product, Tr the trace of an operator, and $Q_J(t)$ the orthographic projection of the space $L_2(\Omega)$ to the subspace spanned by $\{\nu_1(t),\cdots,\nu_J(t)\}$. Therefore, from (4.7) we can obtain that the change of the volume $\wedge_{j=0}^J\xi$ of the J dimensional cube is

$$\begin{aligned}\omega_J(t)&=\sup_{u_0\in A}\sup_{\xi_j\in L_2,|\xi|\le 1}\|\nu_1(t)\wedge\cdots\wedge\nu_J(t)\|^2_{\wedge^J_{L_2}}\\&\le\sup_{u_0\in A}\exp\left(-2\int_0^t\inf(TrL(S_\tau U_0))Q_J(\tau)d\tau\right).\end{aligned}\tag{4.9}$$

From the result in [9] we know that $\omega_j(t)$ is sub-exponented with respect to t, i.e.,

$$\omega_j(t+t')\le\omega_j(t)\omega_j(t'),t,t'\ge 0. \tag{4.10}$$

Hence we have

$$\lim_{t\to\infty}\omega_J(t)^{1/t}=\prod_J\le\exp(-2q_J), \tag{4.11}$$

where

$$q_J=\lim_{t\to\infty}\sup\left(\inf_{u_0\in A,|\xi|\le 1,\xi_j\in C_R}\frac{1}{t}\int_0^t\inf(TrL(S_\tau U_0))Q_J(\tau)d\tau\right). \tag{4.12}$$

Definition 1 *The Hausdorff measure of a set X is defined by*

$$n_H(X,d)=\lim_{\varepsilon\to 0}n_H(X,d,\varepsilon)=\sup_{\varepsilon>0}n_H(X,d,\varepsilon),$$

where

$$n_H(X,d,\varepsilon)=\inf\sum_i r_i^d$$

and the infimum is taken over the balls with radii $r_i\le\varepsilon$ that cover the set X.

The Hausdorff dimension of a set X is defined by a number $d_H(X)\in[0,\infty)$ which satisfies

$$n_H(X,d)=0,\quad\text{for}\quad d>d_H(X)$$

and

$$n_H(X,d)=\infty,\quad\text{for}\quad d<d_H(X).$$

Definition 2 *The fractal dimension is defined by the number*

$$d_F(X)=\lim_{\varepsilon>0}\sup\frac{\lg n_x(\varepsilon)}{\lg\dfrac{1}{\varepsilon}},$$

where $n_x(\varepsilon)$ denotes the smallest number of the balls with radii less than or equal to ε that cover the set X.

From the results of [9] we see that

$$d_F(X) = \inf\{d > 0, n_F(X, d) = 0\}$$

where

$$n_F(X, d) = \lim_{\varepsilon \to 0} \sup(\varepsilon^d n_x(\varepsilon)).$$

Since $n_F(X, d) \geq n_H(X, d)$, we have

Theorem 4.1[10] *Let A be an attractor of a nonlinear evolution equation (such as the Navier Stokes equation (1.7) etc) that is bounded in $H^1(\Omega)$. Then if $q_J > 0$ for some J, the Hausdorff dimension of X is less than or equal to J and its fractal dimension is less than or equal to*

$$J\left(1 + \max_{1 \leq l \leq J} \frac{-q_l}{q_J}\right) \tag{4.13}$$

Lemma 4.1[7] *(A generalization of the Sobolev-Lieb-Thirring inequality) Let $\Omega \subset R^n$ be a bounded domain, and $\{\phi_1, \phi_2, \cdots,\}$ an orthogonal basis in $L_2(\Omega)$. $\phi_i \in H^m$, and for almost every $x \in \Omega$, set $\rho(x) = \sum\limits_{J=1}^{N} |\phi_j|^2$. Then we have the estimate*

$$\int\limits_{\Omega} \rho(x)^{1+\frac{2}{n}} dx \leq \frac{k_0}{|\Omega|^{\frac{2m}{n}}} \sum_{j=1}^{N} \int\limits_{\Omega} |\phi_j|^2 dx + k_0 \sum_{j=1}^{N} \int\limits_{\Omega} |D^m \phi_j|^2 dx \tag{4.14}$$

where the constant k_0 depends on m, n, Ω, but it is independent of N and ϕ_j.

Theorem 4.2 *Under the conditions of Theorem 3.3, the Hausdorff and fractal dimensions of the global attractor of the problem (1.7)-(1.10) are finite, and*

$$d_H(A) \leq J_0, \quad d_F(A) \leq 2J_0, \tag{4.15}$$

where J_0 is the smallest integer which satisfies

$$J_0 \geq (\frac{b}{a})^{1/2},$$

where

$$\begin{aligned} a =& \frac{\min(\alpha, \gamma)}{4k_0 \gamma D^2}, \\ b =& \frac{\min(\alpha, \gamma)}{4D^2} + \frac{1}{2}\|f''(u)\|_\infty + \frac{1}{2}\|g''(v)\|_\infty + \|u_x\|_\infty + \|v_x\|_\infty - b_0. \end{aligned}$$

Proof On account of the result of Theorem 4.1, we need to estimate the lower bound of $Tr\,(L(U)Q_J)$.

Suppose that $\{\phi_1, \phi_2, \cdots, \phi_J\}$ is an orthogonal basis of the subspace $Q_J L_2$, we have

$$\begin{aligned}
Tr\,(L(U)Q_J) =& \sum_{j=1} [(-\alpha\phi_{jxx} - \beta\phi_{jxxx} + (f'(u)\phi_j)_x + 2(v\psi_j)_x - G_{1u}\phi_j - G_{1v}\psi_j, \phi_j) \\
&+(-\gamma\psi_{jxx} + (g'(v)\psi_j)_x + 2(u\phi_j + v\psi_j)_x - G_{2u}\phi_j - G_{2v}\psi_j, \psi_j)] \\
=& \sum_{j=1} \Bigg[\alpha\|\phi_{jx}\|^2 + \gamma\|\psi_{jx}\|^2 + \frac{1}{2}\int_\Omega f''(u)u_x\phi_j^2 dx + \int_\Omega u_x\phi_j^2 dx + \int_\Omega v_x\psi_j^2 dx \\
&+2\int_\Omega v_x\phi_j\psi_j dx - \int_\Omega (G_{1u}\phi_j^2 + G_{1v}\phi_j\psi_j + G_{2u}\phi_j\psi_j + G_{2v}\psi_j^2) dx \\
&+\frac{1}{2}\int_\Omega g''(v)v_x\psi_j^2 dx\Bigg] \\
\geq& \min(\alpha,\gamma)\left[\frac{1}{k_0}\int_\Omega \rho^3(x)dx - \frac{1}{(2D)^2}J\right] + b_0 J \\
&-\left(\frac{1}{2}\|f''(u)\|_\infty + \frac{1}{2}\|g''(v)\|_\infty\|v_x\|_\infty + \|u_x\|_\infty + \|v_x\|_\infty\right) \\
&-\left(\frac{1}{2}\|f''(u)u_x\|_\infty + \frac{1}{2}\|g''(v)v_x\|_\infty + \|u_x\|_\infty + \|v_x\|_\infty\right) J, \qquad (4.16)
\end{aligned}$$

where

$$J = \int_\Omega \rho(x)dx \leq \left(\int_\Omega \rho^3(x)dx)^{1/3}(2D\right)^{2/3}, \qquad (4.17)$$

$$\int_\Omega \rho^3(x)dx \geq \frac{J^3}{(2D)^2}, \qquad (4.18)$$

$$\rho(x) = \sum_{j=1}^{J} |\Phi_j|^2. \qquad (4.19)$$

Hence

$$\begin{aligned}
Tr\,(L(U)Q_J) \geq& \frac{\min(\alpha,\gamma)}{k_0(2D)^2}J^3 + (b_0 - \frac{\min(\alpha,\gamma)}{(2D)^2})J \\
&-\left(\frac{1}{2}\|f''(u)\|_\infty + \frac{1}{2}\|g''(v)\|_\infty + \|u_x\|_\infty + \|v_x\|_\infty\right) J > 0,
\end{aligned}$$

if $J > (\frac{b}{a})^{\frac{1}{2}}$, where

$$\begin{aligned}
a =& \frac{\min(\alpha,\gamma)}{4k_0\gamma D^2}, \\
b =& \frac{\min(\alpha,\gamma)}{4D^2} + \frac{1}{2}\|f''(u)\|_\infty + \frac{1}{2}\|g''(v)\|_\infty + \|u_x\|_\infty + \|v_x\|_\infty - b_0.
\end{aligned}$$

Let $J_0 - 1 \le (\frac{b}{a})^{\frac{1}{2}} \le J_0$.
In view of

$$\frac{q_l}{q_{J_0}} \le \frac{bl - al^3}{aJ_0^3 - bJ_0} \le \frac{2b\sqrt{\frac{b}{3a}}}{3(aJ_0^3 - bJ_0)}$$

from Theoerm 4.1 we finally obtain

$$d_H(A) \le J_0,$$

$$d_F(A) \le J_0 \left(1 + \frac{2b\sqrt{\frac{b}{3a}}}{aJ_0^3 - bJ_0} \right).$$

The proof of the theorem is thus completed.

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