J. Partial Diff. Eqs. **17**(2004), 89–96
©International Academic Publishers Vol.17, No.1

THE GLOBAL WELLPOSEDNESS AND SCATTERING OF THE GENERALIZED DAVEY-STEWARTSON EQUATION

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(Received Sep. 9, 2003)

Abstract We discuss the solution of the Cauchy problem of the generalized Davey-Stewartson equation. When the initial value is small enough, we obtain the global wellposedness of the solution and scattering.
Key Words The generalized Davey-Stewartson equation; the Cauchy problem; scattering.
2000 MR Subject Classification 35L70, 35B10, 37L10.
Chinese Library Classification O175.29.

1. Introduction

In this paper we will prove the global wellposedess and scattering result for the Cauchy problem of the generalized Davey-Stewartson equation when the datum is small enough.In [1], Wang Baoxiang, Guo Boling studied the generalized Davey-Stewartson equation,

$$iu_t + Au = \lambda_1|u|^{p_1}u + \lambda_2|u|^{p_2}u + \mu E(|u|^2)u, \tag{1.1}$$

where $u(t,x)(x=(x_1,x_2,...,x_n))$is a complex function of $(t,x)\in R_+\times R^n$. $\lambda_1,\lambda_2,\mu\in C$,

$$A := \sum_{1\le i,j\le n} a_{ij}\frac{\partial^2}{\partial x_i\partial x_j},$$

$$E(\varphi) = \mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}\varphi. \tag{1.2}$$

In the above, $\mathcal{F}(\mathcal{F}^{-1})$ denotes Fourier (converse) transform, $(a_{ij}),(b_{ij})$ are real invertible matrices satisfying

$$\Big|\sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j\Big| \ge C|\xi|^2, \forall\xi\in R^n, \tag{1.3}$$

In this paper, we will study the initial value problem of the generalized Davey-Stewartson equation with the form :

$$iu_t + Au = \lambda|u|^{2q-2}u + \mu E(|u|^q)|u|^{q-2}u, \tag{1.4}$$

$$u(0,x)=u_0(x), \tag{1.5}$$

where $\lambda,\mu\in$ C, $A, E(\varphi)$are defined in (1.2), respectively.

For any $4/n\le p<\infty$ and $r\in[2,\infty)$,we denote:

$$s(p)=\frac{n}{2}-\frac{2}{p},\quad \frac{2}{\gamma(r)}=n(\frac{1}{2}-\frac{1}{r}),\quad r(p)=\frac{2n(2+p)}{n(2+p)-4}, \tag{1.6}$$

$$\alpha(n)=\begin{cases}\dfrac{2n}{n-2}, & n>2\\ \infty, & n=2\end{cases}. \tag{1.7}$$

Our main result is as follows:

Theorem 1.1 *Suppose $n\ge 2, 2\le q<\infty, s(2q-2)=\frac{n}{2}-\frac{1}{q-1}$,and there exists $\delta_1>0$, such that, when $\|u_0\|_{H^{2q-2}}\le\delta_1$, (1.4),(1.5) has a unique solution satisfying*

$$u\in C\left(0,\infty;H^{s(2q-2)}\right)\bigcap\bigcap_{2<r<\alpha(n)}L^{\gamma(r)}\left(0,\infty;B^{s(2q-2)}_{r,2}\right).$$

Theorem 1.2 *Suppose $n\ge 2, 2\le q<\infty, s(2q-2)=\frac{n}{2}-\frac{1}{q-1}$,and there exists $\delta_1>0$ such that, when $\|u_0\|_{H^{2q-2}}\le\delta_1$,the solution of (1.4)(1.5) has scattering.*

The proof of Theorem 1.2 is omitted.

Let S(t) be a semi-group generated by $i\frac{\partial}{\partial t}+A$. From [2] we can obtain the time-space Strichartz estimate:

$$\|S(t)f\|_{L^{\gamma(r)}(-\infty,\infty;\dot B^s_{r,2})}\le\|f\|_{\dot H^s}, \tag{1.8}$$

$$\left\|\int_0^t S(t-\tau)f(\tau)d\tau\right\|_{L^{\gamma(r)}(0,T;\dot B^s_{r,2})}\le C\|f\|_{L^{\gamma(q)'}(0,T;\dot B^s_{q',2})}, \tag{1.9}$$

where $q,r\in[2,\alpha(n)), 0<T\le\infty$, and C is independent of T. If $f=\sum_{i=1}^I f_i,\quad r,q_i\in[2,\alpha(n)), i=1,2,...,I$, from (1.9) we get:

$$\left\|\int_0^t S(t-\tau)f(\tau)d\tau\right\|_{L^{\gamma(r)}(0,T;\dot B^s_{r,2})}\le C\sum_{i=1}^I\|f_i\|_{L^{\gamma(q_i)'}(0,T;\dot B^s_{q_i',2})}. \tag{1.10}$$

2. The Nonlinear Estimates

Lemma 2.1([1]) *$\forall 1<p<\infty$, we get:*

$$\rho(\xi)=:\frac{\xi_1^2}{\sum_{1\le i,j\le n}b_{ij}\xi_i\xi_j}\in\mathcal{M}_p.$$

where (b_{ij}) satisfies (1.3), $\mathcal{M}_p$ denotes multiplier space.

Lemma 2.2([3]) *Suppose $f(u)$ be a continuous and differentiable k power function, we obtain:*

$$\sum_{|\alpha|=k} D^{\alpha} f(u)=\sum_{p=1}^{k} \sum_{\Lambda_k^p} C(\alpha_1,...,\alpha_p) f^p(u) \prod_{i=1}^{p} D^{\alpha_i}(u), \tag{2.1}$$

$$\begin{aligned}\Big|\sum_{|\alpha|=k} D^{\alpha}(f(u)-f(v))\Big| \leq & C\sum_{p=1}^{k} \sum_{\Lambda_k^p} \Big|f^{(p)}(u)-f^{(p)}(v)\Big| \prod_{i=1}^{p}\Big|D^{\alpha^i}(v)\Big| \\ & + C\sum_{p=1}^{k} \sum_{\Lambda_k^p} \Big|f^{(p)}(u)\Big| \sum_{i=1}^{p}\Big|\prod_{j=1}^{i-1} D^{\alpha^j} u \prod_{j=i+1}^{p} D^{\alpha_j} v D^{\alpha_j}(u-v)\Big|, \end{aligned} \tag{2.2}$$

where $c(\alpha_1,\cdots,\alpha_p)$ is dependent on $\alpha_1,\cdots,\alpha_p$. $\prod_{j=1}^{0}\alpha_j=1, \prod_{p+1}^{p}\alpha_j=1$.

$$\Lambda_k^p=\begin{pmatrix} |\alpha_1|+\cdots+|\alpha_p|=k \\ |\alpha_1|,\cdots,|\alpha_p|\geq 1\end{pmatrix}.$$

Lemma 2.3 *Let $E(\varphi)=\mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}\varphi$. $\triangle_h u(\cdot)=u(\cdot+h)-u(\cdot)$, we have $\triangle_h(E(\varphi))=E(\triangle_h\varphi)$.*

Proof From the definition of $E(\varphi)$and $\triangle_h u$,

$$\begin{aligned}\triangle_h(E(\varphi)) =& \Big(\mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}(\varphi)\Big)(y+h)-\Big(\mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}(\varphi)\Big)(y) \\ =& \int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi(y+h)}\int_{R^n} e^{-i\xi x}\varphi(x)dxd\xi \\ & -\int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi y}\int_{R^n} e^{-i\xi x}\varphi(x)dxd\xi \\ =& \int_{R^n}\int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi y}\left(e^{i\xi h}-1\right)e^{-i\xi x}\varphi(x)dxd\xi. \end{aligned} \tag{2.3}$$

For the same reason:

$$\begin{aligned} E(\triangle_h\varphi) =& \mathcal{F}^{-1}\Big[\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j}\Big]\mathcal{F}(\varphi(x+h)-\varphi(x)) \\ =& \int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi y}\int_{R^n} e^{-i\xi x}(\varphi(x+h)-\varphi(x))dxd\xi \\ =& \int_{R^n}\int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi y}\left(e^{-i(x+h-h)\xi}\varphi(x+h)-e^{i\xi x}\varphi(x)\right)dxd\xi \\ =& \int_{R^n}\int_{R^n}\frac{\xi_1^2}{\sum_{1\leq i,j\leq n} b_{ij}\xi_i\xi_j} e^{i\xi y}e^{-ix\xi}\varphi(x)\left(e^{i\xi h}-1\right)dxd\xi. \end{aligned} \tag{2.4}$$

So we get :$\triangle_h(E(\varphi))=E(\triangle_h\varphi)$.

Lemma 2.4([1]) *Let* $n \geq 2, 4/n \leq p < \infty, r(p) = \dfrac{2n(2+p)}{n(2+p)-4}, 0 \leq s < \infty$, *in addition, if p is not even, we suppose* $[s] \leq p$. *We have:*

$$\||u|^p u\|_{\dot{B}^s_{r(p)',2}} \leq C\|u\|^p_{\dot{B}^{s(p)}_{r(p),2}} \|u\|_{\dot{B}^s_{r(p),2}}. \tag{2.5}$$

Lemma 2.5 *Let* $0 \leq s \leq s(2q-2), n \geq 2, r = r(2q-2)$,*there exists a constant* $C > 0$ *such that*

$$\left\|E(|u|^q)|u|^{q-2}u\right\|_{\dot{B}^s_{r',2}} \leq C\|u\|^{2q-2}_{\dot{B}^{s(2q-2)}_{r,2}} \|u\|_{\dot{B}^s_{r,2}}. \tag{2.6}$$

Proof The first step. When $1 \leq s \leq s(2q-2)$,

$$\left\|E(|u|^q)|u|^{q-2}u\right\|_{\dot{B}^s_{r',2}} = \Big(\int_0^\infty t^{-2v} \sum_{|\alpha|=[s]} \sup_{|h|\leq t} \left\|\triangle_h D^\alpha E(|u|^q)|u|^{q-2}u\right\|^2_{L^{r'}} \frac{dt}{t}\Big)^{1/2}, \tag{2.7}$$

where $v = s - [s]$.
Noticed $D^\alpha(uv) = \sum_{\alpha=\alpha^1+\alpha^2} D^{\alpha^1}uD^{\alpha^2}v$,we have

$$\triangle_h D^\alpha(uv) = \sum_{\alpha=\alpha^1+\alpha^2} \Big[D^{\alpha^1}u_h(\triangle_h D^{\alpha^2}v) + (\triangle_h D^{\alpha^1}u)D^{\alpha^2}v\Big]. \tag{2.8}$$

By virtue of (2.8),

$$\begin{aligned}\left\|\triangle_h D^\alpha E(|u|^q)|u|^{q-2}u\right\|_{L^{r'}} \leq \sum_{\alpha=\alpha^1+\alpha^2} &\Big\|D^{\alpha^1}E(|u_h|^q)\triangle_h D^{\alpha^2}|u|^{q-2}u\Big\|_{L^{r'}} \\ &+ \Big\|\triangle_h D^{\alpha^1}E(|u|^q)D^{\alpha^2}(|u|^{q-2}u)\Big\|_{L^{r'}}.\end{aligned} \tag{2.9}$$

Now we estimate

$$I = \Big\|D^{\alpha^1}E(|u_h|^q)\triangle_h D^{\alpha^2}|u|^{q-2}u\Big\|_{L^{r'}}.$$

From Hölder inequality:

$$I \leq \Big\|D^{\alpha^1}E(|u_h|^q)\Big\|_{L^{1/a_0}} \Big\|\triangle_h D^{\alpha^2}|u|^{q-2}u\Big\|_{L^{1/a_1}}. \tag{2.10}$$

where $a_0 = q(\frac{1}{r} - \frac{s(2q-2)}{n}) + \frac{|\alpha^1|}{n}, a_1 = (q-2)(\frac{1}{r} - \frac{s(2q-2)}{n}) + \frac{1}{r} - \frac{|\alpha^1|}{n}$. It is easy to show $a_0, a_1 > 0, a_0 + a_1 = \frac{1}{r'}$.

From Lemma 2.1, Lemma 2.2 and Hölder inequality,

$$\begin{aligned}\Big\|D^{\alpha^1}E(|u_h|^q)\Big\|_{L^{1/a_0}} \leq& C\Big\|D^{\alpha^1}|u_h|^q\Big\|_{L^{1/a_0}} \\ \leq& C\sum_{p=1}^{|\alpha^1|}\sum_{\Lambda^p_k} \||u|^{q-p}\|_{L^{1/b_0}} \Big\|\prod_{i=1}^p D^{\alpha^1_i}u\Big\|_{L^{1/b_i}} \leq C\|u\|^q_{\dot{B}^{s(2q-2)}_{r,2}},\end{aligned} \tag{2.11}$$

where $b_0 = (q-p)(\frac{1}{r} - \frac{s(2q-2)}{n}), b_i = \frac{1}{r} - \frac{s(2q-2)-|\alpha_i^1|}{n}, i = 1,2,\cdots,p, b_0, b_i > 0, b_0 + \sum_{i=1}^{p} b_i = a_0$, are clear.

From Lemma 2.2,

$$\begin{aligned}\left\|\triangle_h D^{\alpha^2}|u|^{q-2}u\right\|_{L^{1/a_1}} \leq & C\sum_{p=1}^{|\alpha|}\sum_{\Lambda_k^p}\left\{\left\|(|u|^{q-p-2}+|u_h|^{q-p-2})|u-u_h|\prod_{i=1}^{p} D^{\alpha_i^2}u\right\|_{L^{1/a_1}}\right. \\ & \left.+\sum_{i=1}^{p}\left\||u_h|^{q-p-1}\prod_{j=1}^{i-1} D^{\alpha_j^2}u_h \prod_{j=i+1}^{p} D^{\alpha_j^2}uD^{\alpha_i^2}(u_h-u)\right\|_{L^{1/a_1}}\right\}. \qquad (2.12)\end{aligned}$$

First, estimate $I' = \||u|^{q-p-2}|u_h-u|\prod_{i=1}^{p} D^{\alpha_i^2}u\|_{L^{1/a_1}}$ and let

$$\begin{aligned} c_0 &= (q-p-1)(\frac{1}{r} - \frac{s(2q-2)}{n}), \\ c_0' &= \frac{1}{r} - \frac{s-v}{n}, \\ c_i &= \frac{1}{r} - \frac{s(2q-2)-|\alpha_i^2|}{n}, i = 1,2,\cdots,p, \end{aligned}$$

It is easy to verify that $c_0, c_0', c_i (i = 1,2,\cdots,p) > 0$ and $c_0 + c_0' + \sum_{i=1}^{p} c_i = a_1$, by modified Hölder inequality:

$$I' \leq C\|u\|^{q-2}_{\dot{B}^{s(2q-2)}_{r,2}}\|u_h - u\|_{L^{1/c_0'}}. \qquad (2.13)$$

Second,estimate $I'' = \||u|^{q-p-1}\prod_{j=1}^{i-1} D^{\alpha_j^2}u_h \prod_{j=i+1}^{p} D^{\alpha_j^2}uD^{\alpha_i^2}(u_h-u)\|_{L^{1/a_1}}$.Taking

$$\begin{aligned} c_0 &= (q-p-1)(\frac{1}{r} - \frac{s(2q-2)}{n}), \\ c_j &= \frac{1}{r} - \frac{s(2q-2)-|\alpha_j^2|}{n}, j \neq i, j = 1,2,\cdots,p, \\ c_i &= \frac{1}{r} - \frac{s-|\alpha_i^2|-v}{n}, \end{aligned}$$

from modified Hölder inequality and $c_0 + c_i + \sum_{j=1, j\neq i}^{p} c_j = a_1$, ,we get

$$I'' \leq C\|u\|^{q-2}_{\dot{B}^{s(2q-2)}_{r,2}}\|D^{\alpha_i^2}(u_h - u)\|_{L^{1/c_i}}. \qquad (2.14)$$

From(2.11)(2.13)(2.14),we conclude

$$I \leq C\|u\|^{2q-2}_{\dot{B}^{s(2q-2)}_{r,2}}\|u_h - u\|_{L^{1/c_0'}} + C\sum_{i=1}^{p}\|u\|^{2q-2}_{\dot{B}^{s(2q-2)}_{r,2}}\left\|D^{\alpha_i^2}(u_h-u)\right\|_{L^{1/c_i}}. \qquad (2.10')$$

As the same we estimate

$$II = \left\|\triangle_h D^{\alpha^1}E(|u|^q)D^{\alpha^2}(|u|^{q-2}u)\right\|_{L^{r'}}. \qquad (2.15)$$

By virtue of Lemma 2.3 and Hölder inequality,

$$\begin{aligned} II \le & \left\| \triangle_h D^{\alpha^1} E(|u|^q) \right\|_{L^{1/a_0}} \left\| D^{\alpha^2}(|u|^{q-2}u) \right\|_{L^{1/a_1}} \\ \le & C \left\| \triangle_h D^{\alpha^1}(|u|^q) \right\|_{L^{1/a_0}} \left\| D^{\alpha^2}(|u|^{q-2}u) \right\|_{L^{1/a_1}} .. \end{aligned} \tag{2.16}$$

where $a_0 = q(\frac{1}{r} - \frac{s(2q-2)}{n}) + \frac{|\alpha^1| + v}{n}), a_1 = (q-2)(\frac{1}{r} - \frac{s(2q-2)}{n}) + \frac{1}{r} - \frac{s}{n} + \frac{|\alpha^2|}{n}$, with $a_0, a_1 > 0, a_0 + a_1 = 1/r'$.

From Lemma2.2 and Hölder inequality,

$$\begin{aligned} \left\| D^{\alpha^2}(|u|^{q-2}u) \right\|_{L^{1/a_1}} & \le C \sum_{p=1}^{|\alpha^2|} \sum_{\Lambda_k^p} \left\| |u|^{q-p-1} \prod_{i=1}^{p} D^{\alpha_i^2} u \right\|_{L^{1/a_1}} \\ & \le C \sum_{p=1}^{|\alpha^2|} \sum_{\Lambda_k^p} \left\| |u|^{q-p-2} u \right\|_{L^{1/b_0}} \left\| \prod_{i=1}^{p} D^{\alpha_i^2} u \right\|_{L^{1/b_i}} \\ & \le C \|u\|_{\dot{B}_{r,2}^{s(2q-2)}}^{q-2} \|u\|_{\dot{B}_{r,2}^{s}}, \end{aligned} \tag{2.17}$$

where $b_i = \frac{1}{r} - \frac{s(2q-2) - |\alpha_i^2|}{n}, i = 1, 2, \cdots, p$. $b_0 = (q-p-2)(\frac{1}{r} - \frac{s(2q-2)}{n}) + \frac{1}{r} - \frac{s}{n}$. $b_0, b_i > 0, b_0 + \sum_{i=1}^{p} b_i = a_0$.

From Lemma 2.2,

$$\begin{aligned} \left\| \triangle_h D^{\alpha^2} |u|^q \right\|_{L^{1/a_0}} \le & C \sum_{p=1}^{|\alpha|} \sum_{\Lambda_k^p} \left\{ \left\| (|u|^{q-p-1} + |u_h|^{q-p-1}) |u - u_h| \prod_{i=1}^{p} D^{\alpha_i^1} u \right\|_{L^{1/a_0}} \right. \\ & \left. + \sum_{i=1}^{p} \left\| |u_h|^{q-p} \prod_{j=1}^{i-1} D^{\alpha_j^1} u_h \prod_{j=i+1}^{p} D^{\alpha_j^1} u D^{\alpha_i^1}(u_h - u) \right\|_{L^{1/a_0}} \right\}. \end{aligned} \tag{2.18}$$

First, estimate $II' = \left\| |u|^{q-p-1} |u - u_h| \prod_{i=1}^{p} D^{\alpha_i^1} u \right\|_{L^{1/a_0}}$, and let

$$\begin{aligned} c_0 &= (q-p-1)(\frac{1}{r} - \frac{s(2q-2)}{n}), \\ c_0' &= \frac{1}{r} - \frac{s(2q-2) - v}{n}, \\ c_i &= \frac{1}{r} - \frac{s(2q-2) - |\alpha_i^1|}{n}, i = 1, 2, \cdots, p. \end{aligned}$$

$c_0, c_0', c_i (i = 1, 2, \cdots, p) > 0$ and $c_0 + c_0' + \sum_{i=1}^{p} c_i = a_0$.
From modified Hölder inequality:

$$II' \le C \|u\|_{\dot{B}_{r,2}^{s(2q-2)}}^{q-1} \|u_h - u\|_{L^{1/c_0'}}. \tag{2.19}$$

Second, estimate

$$II'' = \left\| |u_h|^{q-p} \prod_{j=1}^{i-1} D^{\alpha_j^1} u_h \prod_{j=i+1}^{p} D^{\alpha_j^1} u D^{\alpha_i^1}(u_h - u) \right\|_{L^{1/a_0}}.$$

Let

$$c_0 = (q-p)(\frac{1}{r} - \frac{s(2q-2)}{n}),$$
$$c_j = \frac{1}{r} - \frac{s(2q-2) - |\alpha_j^1|}{n}, j \neq i, j = 1, 2, \cdots, p,$$
$$c_i = \frac{1}{r} - \frac{s(2q-2) - |\alpha_i^1| - v}{n}.$$

It is clear $c_0, c_j, c_i > 0$ and $c_0 + c_i + \sum_{j=1, j\neq i}^{p} c_j = a_0$.
From modified Hölder inequality:

$$II'' \leq C\|u\|^{q-1}_{\dot{B}^{s(2q-2)}_{r,2}} \|D^{\alpha_i^1}(u_h - u)\|_{L^{1/c_i}}. \tag{2.20}$$

Then from (2.17), (2.19), (2.20)

$$II \leq C\|u\|^{2q-3}_{\dot{B}^{s(2q-2)}_{r,2}} \left(\|u_h - u\|_{L^{1/c_0'}} \|u\|_{\dot{B}^s_{r,2}} + \sum_{i=1}^{p} \left\| D^{\alpha_i^1}(u_h - u) \right\|_{L^{1/c_i}} \|u\|_{\dot{B}^s_{r,2}} \right). \tag{2.15'}$$

So from (2.7), (2.9), (2.10'), (2.15') and Besov space embedding theorem,

$$\|E(|u|^q))|u|^{q-2}u\|_{\dot{B}^s_{r',2}} \leq C\|u\|^{2q-2}_{\dot{B}^{s(2q-2)}_{r,2}} \|u\|_{\dot{B}^s_{r,2}}. \tag{2.21}$$

In the case when $0 \leq s < 1$,the estimation of

$$\left\|E(|u|^q)|u|^{q-2}u\right\|_{\dot{B}^s_{r',2}} = \left(\int_0^\infty t^{-2s} sup_{|h|\leq t} \|\triangle_h(E(|u|^q)|u|^{q-2}u)\|^2_{L^{r'}} \frac{dt}{t} \right)^{1/2}$$

is easy to prove and the proof is omitted.

3. The Proof of Theorem 1.1

(1.4), (1.5) is equal to the integral equation

$$u(t) = S(t)u_0 - i \int_0^t S(t-\tau)F(u(\tau))d\tau \tag{3.1}$$

where $F(u) = \lambda|u|^{2q-2}u + \mu E(|u|^q)|u|^{q-2}u$.

Let $\delta > 0$, define

$$D = \left\{ u \in L^{2q}(0,\infty; B^{s(2q-2)}_{r(2q-2),2}) : \|u\|_{L^{2q}(0,\infty; B^{s(2q-2)}_{r(2q-2),2})} \leq \delta \right\} \tag{3.2}$$

is metric space with

$$d(u,v)=\|u-v\|_{L^{2q}(0,\infty;L^{r(2q-2)})}. \tag{3.3}$$

We consider the mapping:

$$J:u(t)\mapsto S(t)u_0-i\int_0^t S(t-\tau)F(u(\tau))d\tau. \tag{3.4}$$

It is wanted to prove $J:(D,d)\to(D,d)$ is a contractive mapping.

From Lemma 2.4 and Lemma 2.5,we get

$$\begin{aligned}&\big\||u|^{2q-2}u\big\|_{\dot B^{s(2q-2)}_{r(2q-2)',2}}\le C\|u\|^{2q-1}_{\dot B^{s(2q-2)}_{r(2q-2),2}}\\ &\big\|E(|u|^q)|u|^{q-2}u\big\|_{\dot B^{s(2q-2)}_{r(2q-2)',2)}}\le C\|u\|^{2q-1}_{\dot B^{s(2q-2)}_{r(2q-2),2}}.\end{aligned}$$

Since $\dfrac{1}{(2q)'}=\dfrac{2q-1}{2q}$,we have

$$\begin{aligned}&\big\||u|^{2q-2}u\big\|_{L^{(2q)'}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2)',2})}\le C\|u\|^{2q-1}_{L^{2q}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2),2})},\\ &\big\|E(|u|^q)|u|^{q-2}u\big\|_{L^{(2q)'}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2)',2)})}\le C\|u\|^{2q-1}_{L^{2q}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2),2})}.\end{aligned}$$

Therefore, $\forall u\in D$, by virtue of (1.8) and (1.10),

$$\|Ju\|_{L^{2q}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2),2})}\le C\|u_0\|_{\dot H^{s(2q-2)}}+C\delta^{2p-1}.$$

Taking $2C\|u_0\|_{\dot H^{s(2q-2)}}\le\delta, C\delta^{2p-2}=\frac12$, we find

$$\|Ju\|_{L^{2q}(0,\infty;\dot B^{s(2q-2)}_{r(2q-2),2})}\le\frac{\delta}{2}+\frac{\delta}{2}=\delta.$$

Thus $J:D\to D$, and $\forall u,v\in D$. Following the similar way, we get $d(Ju,Jv)\le\dfrac{d(u,v)}{2}$. So from Banach contraction mapping theorem, we obtain a unique fixed point $u\in D$. Thus (1.4) (1.5) has a unique global solution $u\in L^{\gamma(r)}(0,\infty;B^{s(2q-2)}_{r,2})$, $(\forall r\in[2,\alpha(n)))$ is an immediate consequence of Strichartz estimate. The proof of Theorem 1.1 is completed.

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