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NON-NEGATIVE RADIAL SOLUTION FOR AN ELLIPTIC EQUATION

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Abstract We study the structure and behavior of non-negative radial solution for the following elliptic equation

$$\Delta u = u^\nu, \quad x \in \mathbb{R}^n$$

with $0 < \nu < 1$. We also obtain the detailed asymptotic expansion of u near infinity.
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1. Introduction

In this paper, we consider the structure and behavior of non-negative radial solution of the following nonlinear equation

$$\Delta u = u^\nu, \quad x \in \mathbb{R}^n, \quad 0 < \nu < 1. \tag{1.1}$$

Problem (1.1) appears in several applications in mechanics and physics, and in particular can be treated as the equation of equilibrium states in thin films. For backgrounds on (1.1), we refer to [1 - 7] and the references therein.

The Cauchy problem

$$\begin{cases} u_t = \Delta u + u^p, \quad x \in \mathbb{R}^n, \quad t > 0, \quad p > 1, \\ u|_{t=0} = \phi \in C_0(\mathbb{R}^n) \equiv C(\mathbb{R}^n) \cap L_\infty(\mathbb{R}^n), \quad \phi \geq 0, \ \ \phi \not\equiv 0 \end{cases} \tag{1.2}$$

has been studied by many authors ([8 - 12]). The structure and expansion of the non-negative radial solution of the steady-state problem of (1.2) also have been studied in [12, 13]. The author also refers to [14, 15] when this paper is in preparation.

2. Structure and Behavior

Definition 2.1 *We say that u is a regular solution of (1.1) if $u \in C^2(\mathbb{R}^n)$ and u satisfies (1.1). We call u a singular solution of (1.1) if $u \in C^2(\mathbb{R}^n \setminus \{0\}) \cap C(\mathbb{R}^n)$ satisfies (1.1) in $\mathbb{R}^n \setminus \{0\}$ with non-removable zero at $x = 0$.*

In the follows, we set

$$\delta = \frac{2}{1-\nu}, \qquad L = \Big[\delta(\delta + n - 2)\Big]^{\frac{1}{\nu-1}}.$$

Proposition 2.2. *When $0 < \nu < 1$, all nontrivial non-negative radial regular solutions of (1.1) are included in a family $\{u_\alpha\}_{\alpha>0}$ with u_α being the unique positive solution of the problem*

$$\begin{cases} u'' + \dfrac{n-1}{r} u' = u^\nu \quad in \ \ (0, \infty), \\ u(0) = \alpha, \qquad u'(0) = 0. \end{cases} \tag{2.1}$$

u_α is increasing in r, $r^{2/(\nu-1)} u_\alpha(r) \to L$ as $r \to \infty$ and $u_\alpha(r) = \alpha u_1(\alpha^{(\nu-1)/2} r)$. Moreover, the only radial singular solution of (1.1) is

$$u_0(r) = L r^{2/(1-\nu)}.$$

Proof We can obtain the result by phase plane analysis, see [16]. Assume u is a nontrivial non-negative radial solution of (1.1). Let

$$r = |x|, \quad t = -\ln r, \quad v(t) = r^{-\delta} u(r). \tag{2.2}$$

By u_r and u_{rr}, then we have

$$u_{rr} + \frac{n-1}{r} u_r = r^{\delta\nu} v^\nu,$$

i.e.,

$$v'' - (2\delta + n - 2)v' + (\delta^2 + n\delta - 2\delta)v = v^\nu. \tag{2.3}$$

Let $q(v) = v'(t)$, then $v_{tt} = \dfrac{dq}{dv} q$. Denote $C_0 = 2\delta + n - 2$ and by (2.3) we have

$$q\frac{dq}{dv} - C_0 q + v(L^{\nu-1} - v^{\nu-1}) = 0. \tag{2.4}$$

On (v, v_t) plane, we know $(L, 0)$ is the only unstable equilibrium point, which implies $v(t) \to L$ as $t \to -\infty$. That is

$$\lim_{r\to\infty} r^{-2/(1-\nu)} u(r) = L.$$

If $u_\alpha(0) = \alpha > 0$, by scaling invariance, we have $u_\alpha(r) = \alpha u_1(\alpha^{(\nu-1)/2} r)$. All solutions of (1.1) form a one-parameter family of solutions.

We can obtain that $r^{n-1}u_r$ is increasing in r since $(r^{n-1}u_r)_r = r^{n-1}u^\nu \geq 0$. It follows that $r^{n-1}u_r \geq 0$. So u is increasing in r.

If $u(0) = 0$, from above we know $u_r \geq 0$ and $r^{n-1}u_r \to 0$ as $r \to 0$. Noting that

$$r^{n-1}u_r = \int_0^r s^{n-1}u^\nu(s)ds \leq u^\nu(r)\int_0^r s^{n-1}ds = \frac{1}{n}u^\nu(r)r^n,$$

we have $u_r u^{-\nu}(r) \leq \dfrac{r}{n}$. Now integrating over $(0, r)$, we see that

$$\int_0^r u_r u^{-\nu}(s)ds \leq \int_0^r \frac{s}{n}ds,$$

i.e.,

$$u(r) \leq Cr^\delta, \tag{2.5}$$

where C is a constant depending on n and ν. We consider the function v defined at (2.2) which satisfies (2.3) and then we know $v(t) \to L$ as $t \to -\infty$. From (2.5), we see that $v(t) \leq C$ for all t. A simple ODE theory shows that $v(t) \to L$ as $t \to +\infty$ (since L is the only positive equilibrium point).

Now multiplying the equation of (2.3) for $v(t)$ by $v'(t)$ and integrating over $(-\infty, +\infty)$, we see that

$$\int_{-\infty}^{+\infty} v''v' - (2\delta + n - 2)(v')^2 + \delta(\delta + n - 2)vv'dt = \int_{-\infty}^{+\infty} v^\nu v'dt.$$

Noting the fact $v(t) \to L$ as $t \to -\infty$ and $t \to \infty$, we obtain

$$-(n + 2\delta - 2)\int_{-\infty}^{+\infty} (v'(t))^2dt = 0,$$

which implies $v(t) \equiv L$. Thus $u = u_0(r) = Lr^{2/(1-\nu)}$.

Proposition 2.3 *Assume $\bar{u}$ ($\underline{u}$) is a radial regular super-solution (sub-solution) of (1.1). If u_α is a positive radial regular solution of (1.1), then for any $\theta > (<)1$, $\bar{u}$ ($\underline{u}$) cannot stay above (below) θu_α.*

Proof Suppose that $\bar{u} > \theta u_\alpha$, let $v(t) = \dfrac{\bar{u}(r)}{u_\alpha(r)}$, $t = \ln r$, then $v > 1$ and

$$v''(t) + \left(\frac{2ru'_\alpha(r)}{u_\alpha(r)} + n - 2\right)v'(t) + r^2u_\alpha^{\nu-1}(r)(v - v^\nu)(t) < 0 \quad on\ (-\infty, \infty). \tag{2.6}$$

Denote the coefficient of v' by $g_1(t)$. By (2.6) and the fact $v > 1$, $v'' + g_1(t)v' < 0$. Hence

$$\exp\left\{\int_0^t g_1(s)ds\right\}v'(t) \leq \exp\left\{\int_0^\tau g_1(s)ds\right\}v'(\tau) \quad \text{if } \ t \geq \tau. \tag{2.7}$$

In fact, $(\exp\{\int_0^t g_1(s)ds\}v'(t))' < 0$. Since $ru'_\alpha(r) \to 0$ as $r \to 0^+$, we have $g_1(t) \to n - 2$ as $t \to -\infty$. It follows from the fact $v(t) \to \dfrac{\bar{u}(0)}{u_\alpha(0)}$ as $t \to -\infty$ that there exists a sequence $t_m \to -\infty$ such that $v'(t_m) \to 0$. Now in (2.7), letting $\tau = (t_m) \to -\infty$, we have either $v' < 0$ on $(-\infty, +\infty)$) or $v' \equiv 0$(A priori, $v'(t) \le 0$ and if there exists t_0 such that $v'(t_0) \le 0$, then by (2.7) again $v'(t) < 0$ if $t \ge t_0$. Hence the strict inequality in (2.7) must be true which in turn implies that $v' < 0$ on $(-\infty, +\infty)$. But by (2.6) and $v > 1$ we see that $v' \equiv 0$ is impossible. Since $v' < 0$ we have for a large T and some constant $C > 0$,

$$v'' + g_1(t)v' \le -C \qquad if \quad t \ge T,$$

which implies that $v = 0$ at some t_0. This contradicts the fact $v > 1$.

Another case can be proved similarly.

3. Expansion Near Infinity

In this section, we study the expansion of u near infinity which is the non-negative radial solution of (1.1) for $n \ge 3$.

Noting that $\delta = \dfrac{2}{1-\nu} \in (2, \infty)$, we have

$$(2\delta + n - 2)^2 - 8(\delta + n - 2) > 0.$$

Therefore, the solutions of the equation

$$\sigma_{tt} + (2\delta + n - 2)\sigma_t + 2(\delta + n - 2)\sigma = 0 \tag{3.1}$$

can be written as linear combinations of $e^{-\lambda_1 t}$ and $e^{-\lambda_2 t}$, where

$$\lambda_1(\nu, n) = \frac{2\delta + n - 2 - [(2\delta + n - 2)^2 - 8(\delta + n - 2)]^{1/2}}{2} > 0 \tag{3.2}$$

and

$$\lambda_2(\nu, n) = \frac{2\delta + n - 2 + [(2\delta + n - 2)^2 - 8(\delta + n - 2)]^{1/2}}{2} > 0 \tag{3.3}$$

are two roots of

$$\lambda^2 - (2\delta + n - 2)\lambda + 2(\delta + n - 2) = 0. \tag{3.4}$$

To study the behavior of the solutions of (3.1), we consider three cases: (a) $2\delta + n - 2 - 2\lambda_1 < \lambda_1$, (b) $2\delta + n - 2 - 2\lambda_1 = \lambda_1$, (c) $2\delta + n - 2 - 2\lambda_1 > \lambda_1$.

If (a) occurs, we have

$$\begin{aligned} 2\delta + n - 2 - 2\lambda_1 &= [(2\delta + n - 2)^2 - 8(\delta + n - 2)]^{1/2} \\ &< \frac{1}{2}(2\delta + n - 2) - \frac{1}{2}[(2\delta + n - 2)^2 - 8(\delta + n - 2)]^{1/2}, \end{aligned}$$

i.e.,

$$(2\delta + n - 2)^2 < 9(\delta + n - 2)$$

and

$$4\delta^2 + (4n - 17)\delta + (n-2)(n-11) < 0. \tag{3.5}$$

This equation has two roots:

$$\delta_1 = \frac{17 - 4n - 3(8n-7)^{1/2}}{8},$$

$$\delta_2 = \frac{17 - 4n + 3(8n-7)^{1/2}}{8}.$$

We easily know that

$$\delta_1 < \delta < \delta_2.$$

So, if $\delta_1 < \delta < \delta_2$, then $2\delta + n - 2 - 2\lambda_1 < \lambda_1$. Thus we know from (4.28) of [15] that any solution $\sigma(t)$ of (3.1) satisfies

$$\sigma(t) = \begin{cases} a_1 e^{-\lambda_1 t} + O(e^{-\lambda_2 t}) & \text{if} \quad \delta_1 < \delta < \delta_2, \\ a_1 e^{-\lambda_1 t} + O(te^{-2\lambda_2 t}) & \text{if} \quad \delta = \delta_2, \\ a_1 e^{-\lambda_1 t} + O(e^{-2\lambda_2 t}) & \text{if} \quad \delta > \delta_2. \end{cases} \tag{3.6}$$

It is straightforward to show that for $n \geq 3$ there exists an infinite sequence $\nu_k < \nu_{k+1} < \cdots < 1$ such that $\lambda_2(\nu, n) = k\lambda_1(\nu, n)$ if and only if $\nu = \nu_k(n)$, where [a] = the largest integer which is smaller than $a + 1$. It is not hard to see that

$$\nu_k = \frac{n + 2 - z_k}{n - 2 - z_k}, \qquad k \geq \left[\frac{n}{2}\right],$$

where z_k is the only zero of $h(z) - k = 0$ and the function

$$h(z) = \frac{[z + (z^2 - 4z - 4(n-2))^{1/2}]^2}{4(z + n - 2)}, \qquad z \in [n+2, \infty)$$

is strictly increasing in $[n+2, \infty)$. It is also possible to give a more explicit expression for $\nu_k(n)$. To this end we set $y = 2\delta + n - 2 > n + 2$. Then $\lambda_2 = k\lambda_1$ if and only

$$k = \frac{\lambda_2}{\lambda_1} = \frac{y + (Y(y))^{1/2}}{y - (Y(y))^{1/2}},$$

which is equivalent to

$$\frac{k-1}{k+1} = \frac{(Y(y))^{1/2}}{y} = \left[1 - \frac{4}{y} - \frac{4(n-2)}{y^2}\right]^{1/2}, \tag{3.7}$$

where Y is defined

$$Y(y) = y^2 - 4y - 4(n-2).$$

Squaring both side of (3.7) and multiplying by y^2, we obtain

$$\left[1-\left(\frac{k-1}{k+1}\right)^2\right]y^2-4y-4(n-2)=0. \tag{3.8}$$

Now $\nu_k(n)$ may be obtained by solving y explicitly(in fact, we can obtain $z_k = y$). Incidentally, the fact that $k \geq \frac{n}{2}$ also follows easily from (3.7) since $y > n+2$ and then

$$\frac{k-1}{k+1} \geq \frac{(Y(n+2))^{1/2}}{n+2} = \frac{n-2}{n+2}.$$

Thus, $k \geq \frac{n}{2}$.

It follows from Proposition 2.2 that if u is a non-negative radial solution of (1.1), then $\lim\limits_{r\to\infty} r^{-\delta}u(r)$ must always exist. Now we derive a more detailed asymptotic expansion of u near infinity.

Theorem 3.1 *Let u be a non-negative radial solution of (1.1) and $\lim\limits_{r\to\infty} r^{-\delta}u(r) > 0$. Then the following statements hold:*

(i) *For $\nu = \nu_k(n)$, $k \geq \left[\frac{n}{2}\right]$, we have $\lambda_2 = k\lambda_1$ and, near infinity,*

$$\begin{aligned} u(r) = {} & Lr^\delta + a_1 r^{\delta-\lambda_1} + \cdots + a_{k-1}r^{\delta-(k-1)\lambda_1} \\ & + a_k r^{\delta-k\lambda_1}\ln r + br^{\delta-\lambda_2} + \cdots + O(r^{-(n+2-\epsilon)}). \end{aligned} \tag{3.9}$$

(ii) *For $\nu_k < \nu < \nu_{k+1}(n)$, $k \geq \left[\frac{n}{2}\right]$, we have $k\lambda_1 < \lambda_2 < (k+1)\lambda_1$, and, near infinity,*

$$\begin{aligned} u(r) = {} & Lr^\delta + a_1 r^{\delta-\lambda_1} + \cdots + a_k r^{\delta-k\lambda_1} \\ & + br^{\delta-\lambda_2} + cr^{\delta-(k+1)\lambda_1} + \cdots + O(r^{-(n+2-\epsilon)}). \end{aligned} \tag{3.10}$$

The constant $L = [\delta(n+\delta-2)]^{1/(\nu-1)}$ and is independent of the particular solution u. The coefficients a_2, a_3, $\cdots$, a_n, $\cdots$ are uniquely determined once a_1 is determined. Moreover, once a_1, b are determined then all the coefficients in (3.9) and (3.10) are uniquely determined.

Proof We start with the proof of (ii). The proof is closely related to the proof of Theorem 2.5 of [3, 9]. First , we know from Proposition 2.2 that

$$\lim_{r\to\infty} r^{-\delta}u(r) = L.$$

Setting $t = \ln r$, $w(t) = r^{-\delta}u(r) - L$, we see that w satisfies the equation

$$w_{tt} + (2\delta+n-2)w_t + 2(\delta+n-2)w(t) - g(w) = 0, \quad t \geq t_0 = \ln R \tag{3.11}$$

and $g(\tau) = (\tau+L)^\nu - L^\nu - \nu L^{\nu-1}\tau$ satisfies

$$g(\tau) = \frac{\nu(\nu-1)}{2}L^{\nu-2}\tau^2 + O(\tau^3) \quad \text{for } \tau \text{ near } 0. \tag{3.12}$$

By standard arguments it follows that

$$w(t) = a_1 e^{-\lambda_1 t} + b e^{-\lambda_2 t} + \frac{1}{\lambda_1 - \lambda_2} \int_{t_0}^{t} \left(e^{\lambda_2 (s-t)} - e^{\lambda_1 (s-t)} \right) g(w(s)) dt', \tag{3.13}$$

where a_1, b are two constants. Notice that $-\lambda_1$, $-\lambda_2$ are the roots of the characteristic polynomial of the linear part of (3.11), where λ_1 and λ_2 are in (3.2) and (3.3). For each positive integer $M \geq 2$, $g(\tau)$ admits the following expansion

$$g(\tau) = d_2 \tau^2 + d_3 \tau^3 + \cdots + d_M \tau^M + O(\tau^{M+1}) \tag{3.14}$$

near $\tau = 0$, where the constants $d_2, d_3, \cdots, d_M$ depend only upon ν and n. When $k \geq \left[\frac{n}{2}\right]$, $(n \geq 3)$, we have from (3.6) that (since $\lambda_2 > 2\lambda_1$, $2\delta + n - 2 - 2\lambda_1 > \lambda_1$)

$$w(t) = a_1 e^{-\lambda_1 t} + O(e^{-2\lambda_1 t}) \tag{3.15}$$

near $t = \infty$. Substituting (3.14) and (3.15) into (3.13) we obtain

$$\begin{aligned} w(t) &= a_1 e^{-\lambda_1 t} + b e^{-\lambda_2 t} + \frac{1}{\lambda_1 - \lambda_2} \int_{t_0}^{t} (e^{\lambda_2 (s-t)} - e^{\lambda_1 (s-t)}) \varphi(s) ds \\ &= a_1 e^{-\lambda_1 t} + b e^{-\lambda_2 t} + \frac{1}{\lambda_1 - \lambda_2} \left\{ \int_0^t e^{\lambda_2 (s-t)} \varphi(s) ds - \int_0^{t_0} e^{\lambda_2 (s-t)} \varphi(s) ds \right\} \\ &\quad - \frac{1}{\lambda_1 - \lambda_2} \left\{ \int_{t_0}^{\infty} e^{\lambda_1 (s-t)} \varphi(s) ds - \int_t^{\infty} e^{\lambda_1 (s-t)} \varphi(s) ds \right\} \\ &= a_1 e^{-\lambda_2 t} + a_2 e^{-2\lambda_1 t} + O(e^{-\min(3\lambda_1, \lambda_2) t}), \end{aligned} \tag{3.16}$$

where $\varphi(s) = d_2 a_1^2 e^{-2\lambda_1 s} + O(e^{-(\lambda_1 + 2\lambda_1) s})$, positive constant a_2 depends only on a_1, ν and n. Substituting (3.16) and (3.14) into (3.13) , by similar computation , after $(k-1)$ steps we arrive at

$$w(t) = a_1 e^{-\lambda_1 t} + a_2 e^{-2\lambda_1 t} + \cdots + a_k e^{-k\lambda_1 t} + O(e^{-\lambda_2 t}) \tag{3.17}$$

near infinity, where $a_2 = a_2(a_1, \nu, n)$, $a_k = a_k(a_1, \nu, n)$. Repeating this process once more, we obtain

$$w(t) = a_1 e^{-\lambda_1 t} + a_2 e^{-2\lambda_1 t} + \cdots + a_k e^{-k\lambda_1 t} + b_1 e^{-\lambda_1 t} + O(e^{-(k+1)\lambda_1 t}) \tag{3.18}$$

near infinity. Now iterating the above process with (3.18), (3.14) into (3.13) after finitely many steps (with the integer M in (3.14) getting larger each time) we arrive at, for each positive integer ℓ,

$$w(t) = \sum_{i=1}^{\ell+k} a_i e^{-i\lambda_1 t} + \sum_{j \in J} b_j e^{-j\lambda_2 t} + \sum_{(i,j) \in I} c_{ij} e^{-(i\lambda_1 + j\lambda_2) t} + O(e^{-(\ell\lambda_1 + \lambda_2) t}), \tag{3.19}$$

where

$$J = \{j \in Z : j \geq 1, j\lambda_2 < \ell\lambda_1 + \lambda_2\},$$
$$I = \{(i,j) \in Z \times Z : \ i \geq 1, j \geq 1, i\lambda_1 + j\lambda_2 < \ell\lambda_1 + \lambda_2\}$$

and a_i depend only upon a_1, ν, n, b_j depend only upon b_1, ν, n, and c_{ij} depend only upon a_1, b_1, ν, n. (Here Z=the set of all integers.) Taking ℓ large enough and then substituting (3.19) into $w(t) = r^{-\delta}u(r) - L$, we obtain (3.10).

Part(i) may be proved similarly by the arguments above together with the proof of Lemmas 4.3 and 4.4 in [15].

Remark Theorem 3.1 is stated in a special way with the forms of expansions (3.9) and (3.10). The expansion of u near infinity may have more general forms. In particular, it is clear from the proof above what the missing terms in (3.9) and (3.10) are. Moreover, it is also clear from the proof above that the expansions (3.9) and (3.10) do not have to stop at $O(r^{-(n+2-\epsilon)})$, they can go on to an arbitrarily high order.

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