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Investigation on Effective Heat Conductivity of Fibrous Assemblies by Fractal Method

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Abstract

A fractal model for predicting the effective heat conductivity of fibrous assemblies was established. In the model, fiber-to-fiber contact influence on solid heat conductivity was taken into consideration. Radiative heat conductivity was also considered to get the effective heat conductivity. The effective heat conductivity was proved to be related to the following parameters, including the pore area fractal dimension, tortuosity fractal dimension, maximum and minimum pore diameters, solid conductivity, air conductivity, porosity and the ratio of the number of perpendicular channels to the total number of channels. Experiment was conducted to verify the model, and good agreement was found between the experimental and theoretical results.

Keywords: Effective Heat Conductivity; Fibrous Assembly; Fractal; Fiber-to-fiber Contact; Radiative Heat Conductivity

1 Introduction

The study of the effective heat conductivity of fibrous assemblies has received continuous attention [1-3] due to their various applications in clothing and engineering. Numerous researchers have worked on the effective heat conductivity calculation of fibrous assemblies [4-6]. These studies are all based on the assumption that the fibrous assemblies are a continuous medium, which makes it difficult to consider the influence of microstructure of pores, and thus the application of these existing theories has some fundamental limitations.

Fractal theory has been applied to study the thermal conductivity of porous media [7-9]. Chen et al [8] developed a fractal model to study effective heat conductivity of soil. Later, Yu et al [9] proposed fractal models to calculate the effective thermal conductivity of mono- and bi-dispersed porous media, such as sandstone and particles etc. The effective heat conductivity of wood [10], foam [11] and other objects were also discussed by some researchers [12, 13]. The above models

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established were based on the assumption that the objects investigated are exactly self-similar, which is not in accordance with real porous media. For real porous media, the microstructure are statistically self-similar. Kou et al [14] investigated the effective heat conductivity of fibrous materials under this condition. The model assumed that the air and fibers are in parallel arrangement in the fibrous materials, which neglects the fiber-to-fiber contact effect on the effective heat conductivity. The previous models also did not take radiative heat conductivity into consideration.

In this paper, we will use the fractal theory to calculate the effective heat conductivity of fibrous assemblies. Fiber-to-fiber contact effect on the property was taken into consideration. What is more, radiative heat conductivity was also considered. Experiment is conducted to verify the model.

2 The Effective Heat Conductivity Calculation by Fractal Method

2.1 Microstructure and Fractal Description of Porous Media

As is known, an object measurement is related to its dimension and is invariant with the unit of measurement used. In general, ordered objects such as points, lines, surfaces and cubes can be described by Euclidean geometry using integer dimension 0, 1, 2 and 3, respectively. However, it is found that numerous objects in nature, such as rough surfaces, coastlines, mountains, rivers, lakes and islands are disordered and irregular, and they cannot be described by the Euclidean geometry because of the scale-dependent measures of length, area and volume. These objects are called fractals, and the dimensions of such objects are non-integral and defined as fractal dimensions. A fractal object measurement $M(L)$ is related to the length scale$L$ by the following power form [15].

$$M(L) \sim L^{D_f} \tag{1}$$

Where the "∼" should be read as "scale as". M can be the length of a line, the area of a surface, the volume of a cube, or the mass of an object. D_f is the fractal dimension of the object, $0 < D_f < 2$ in two dimensions. For real porous media, the size distribution of pores satisfies the fractal power law [8, 9],

$$N(L \geq p_{\min}) = \left(\frac{p_{\max}}{p}\right)^{D_f} \tag{2}$$

Where p, $p_{\min}$ and $p_{\max}$ is the pore size, minimum pore size and maximum pore size respectively.

The number of pores within the infinitesimal range p to $p+dp$ can be deduced by a differentiating equation (2) with respect to p.

$$-dN = D_f p_{\max}^{D_f} p^{-(1+D_f)} dp \tag{3}$$

Dividing Eq. 3 by 2, Eq. 4 is obtained:

$$\frac{-dN}{N} = D_f p_{\min}^{D_f} p^{-(1+D_f)} dp = f(p)dp \tag{4}$$

In the above equation, $f(p)$ is the probability density function and should satisfy the following relationship (5).

$$\int_0^\infty f(p)dp = \int_{p_{\min}}^{p_{\max}} f(p)dp = 1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_f} \equiv 1 \tag{5}$$

It is obvious that equation (5) holds if equation (6) is satisfied.

$$\frac{p_{\min}}{p_{\max}} \cong 0 \tag{6}$$

That is to say $p_{\min} \ll p_{\max}$ in equation (6) must be satisfied for fractal analysis of porous media. In general, $p_{\min}/p_{\max} \leq 10^{-2}$ in porous media, so the fractal theory can be used to study the characters of porous media.

It is considered that a unit cell in a fibrous assembly includes a bundle of tortuous capillary tubes with variable cross-sectional area. Let the diameter of a capillary in the fibrous assembly be p and its tortuous length along the flow direction be $L(p)$. Due to the tortuous nature of the capillary, $L(p) \geq L_0$, with L_0 being the representative length. The equation, $L(p) = L_0$, holds for a straight capillary. When heat flows through the pores of the fibrous assembly, the capillaries may be tortuous. These tortuous capillaries can be described by fractal equation [9]:

$$\frac{L(p)}{L_0} = \left(\frac{L_0}{p}\right)^{D_t - 1} \tag{7}$$

Where D_t is the tortuosity fractal dimension, in the range of $1 \leq D_t \leq 2$, which represents the extent of convolutions of capillary pathways for heat flow through a medium. The higher the value D_t, the higher the tortuous capillary. For a straight capillary path, $D_t = 1$, the limiting case of $D_t = 2$ corresponds to a highly tortuous line that fills a plane.

2.2 The Fractal Model for the Effective Heat Conductivity

Heat transfer through fibrous materials involves combined modes of heat transfer: solid conduction through fibers, air conduction and natural convection in the space between fibers, and radiation interchange through participating media. Natural convection heat transfer in fibrous materials with densities greater than 20 kg/m^3 is negligible [16]. According to Stark and Fricke [17], the total heat flux (q_{sa}) passing through a fibrous material is:

$$q_{total} = q_{sa} + q_r \tag{8}$$

Where q_{sa} is the heat flux by solid fibers and air, and q_r is the heat flux due to radiation.

By using Fourier's law $q = -\lambda$ grad T, the effective heat conductivity is:

$$k_{eff} = k_{sa} + k_r \tag{9}$$

So the effective heat conductivity can be calculated from the derivation of solid and air heat conductivity and the radiative heat conductivity.

2.2.1 Heat Conductivity by Solid and Air (k_{sa})

In general, numerous capillary channels are both parallel and perpendicular to heat flow direction for real fibrous assemblies, so the heat conductivity by solid and air can be divided into two parts, which are parallel and perpendicular heat conductivities k_{par}, k_{per}. The total heat conductivity by solid and air can be expressed as:

$$k_{sa} = \delta k_{per} + (1-\delta)k_{par} \tag{10}$$

Where δ is the ratio of the number of perpendicular channels to the total number of channels, with values ranging from 0 to 1.

2.2.1.1 Fractal Parallel Model k_{par}

By heat-electric analogy approach, the parallel heat conductivity can be calculated by assuming that fibers and air channels are in parallel arrangement to heat flow.

According to Fourier's law, the thermal resistance of a single channel r can be expressed as:

$$r(p) = \frac{L(p)}{Ak} \tag{11}$$

Where k is heat conductivity. The thermal resistance of a single air channel can be expressed as:

$$r_a(p) = \frac{L(p)}{A_a k_a} = \frac{4L(p)}{\pi p^2 k_a} \tag{12}$$

The heat resistance of the air channels with the diameter between p and p+dp can be written as:

$$R_{-dN}(p) = \frac{r_a(p)}{-dN} = \frac{4L(p)}{\pi p^2 k_a D_f p_{\max}^{D_f} p^{-(D_f+1)} dp} \tag{13}$$

Substituting Eq. 7 into Eq. 13, we can get Eq. 14.

$$R_{-dN}(p) = \frac{r_a(p)}{-dN} = \frac{4L_0^{D_t}}{\pi p^2 k_a D_f p_{\max}^{D_f} p^{D_t-D_f} dp} \tag{14}$$

According to heat-electrical analogy principle, the total heat resistance of air phase can be described as:

$$\begin{aligned} R_a(p) &= \frac{1}{\sum\limits_{p_{\min}}^{p_{\max}} \frac{1}{R_{-dN}(p)}} = \frac{4L_0^{D_t}}{\int\limits_{p_{\min}}^{p_{\max}} \pi k_a D_f p_{\max}^{D_f} p^{D_t-D_f} dp} \\ &= \frac{4L_0^{D_t}(D_t - D_f + 1)}{\pi k_a D_f p_{\max}^{D_t+1}\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_t-D_f+1}\right]} \end{aligned} \tag{15}$$

The heat transfer resistance caused by fibers can be written as

$$R_s = \frac{L_0}{(1-\varepsilon)Ak_s} \tag{16}$$

Theoretical modeling of solid conduction through fibers and points of contact between them is difficult, and various empirical relations have been developed to model the solid conduction. The empirical model used in this study was:

$$k_s = (1-\varepsilon)^m k_s^* \tag{17}$$

Where m is the constant determined by comparing the experimental and theoretical results.

The total parallel heat conductivity can be written as:

$$k_{par} = \frac{L_0}{A}\left(\frac{1}{R_a} + \frac{1}{R_s}\right) = \frac{\pi D_f p_{\max}^{D_t+1}\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_t - D_f + 1}\right]}{4A(D_t - D_f + 1)L_0^{D_t-1}} k_a + (1-\varepsilon)k_s \tag{18}$$

Where A is the surface area and can be expressed as:

$$A = \frac{A_p}{\varepsilon} = -\frac{1}{\varepsilon}\int_{p_{\min}}^{p_{\max}} \frac{1}{4}\pi p^2 dN = \frac{\pi D_f p_{\max}^2}{4(2-D_f)\varepsilon}\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{2-D_f}\right] \tag{19}$$

Therefore, the total parallel heat conductivity can be obtained:

$$k_{par} = \frac{L_0}{A}\left(\frac{1}{R_a} + \frac{1}{R_s}\right) = \frac{(2-D_f)\varepsilon\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_t - D_f + 1}\right]}{(D_t - D_f + 1)\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{2-D_f}\right]}\left(\frac{p_{\max}}{L_0}\right)^{D_t-1} k_a + (1-\varepsilon)k_s \tag{20}$$

2.2.1.2 Fractal Perpendicular Model

The perpendicular heat conductivity can be calculated by assuming that fibers and air channels are in perpendicular arrangement to heat flow. The perpendicular heat conductivity can be written as:

$$k_{per} = \frac{1}{\frac{\varepsilon}{k_a} + \frac{1-\varepsilon}{k_s}} \tag{21}$$

Inserting Eq. 20 and Eq. 21 into Eq. 10, we can obtain:

$$k_{fa} = (1-\delta)\left\{\frac{(2-D_f)\varepsilon\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_t - D_f + 1}\right]}{(D_t - D_f + 1)\left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{2-D_f}\right]}\left(\frac{p_{\max}}{L_0}\right)^{D_t-1} k_a + (1-\varepsilon)k_f\right\} + \delta\frac{1}{\frac{1}{k_a} + \frac{1-\varepsilon}{k_f}} \tag{22}$$

2.2.2 Radiative Heat Conductivity k_r

The radiative heat conductivity can be obtained from the following equation, according to the studies by previous researchers [18,19,20].

$$k_r = C\sigma T^3 \frac{R'}{e(1-\varepsilon)} \tag{23}$$

Where R' is the radius of fiber, e is the emissivity of the fiber, σ is the Boltzmann constant, 5.67×10^{-8} w/m^{2}k^4, T is the temperature and C is the constant determined by fiber orientation.

In the previous study, we found the new constant by comparing theoretical model and experiment results, which is more accurate [21].

$$k_r = 3.315\sigma T^3 \frac{R'}{e(1-\varepsilon)} \tag{24}$$

2.2.3 The Total Effective Heat Conductivity

The total effective heat conductivity can be obtained by substituting Eq. 22 and Eq. 24 into Eq. 9.

$$k_{sa} = \frac{X}{\delta \dfrac{\varepsilon k_s + (1-\varepsilon)k_a}{k_a k_s} X + (1-\delta)Y} + 3.315\sigma T^3 \frac{R'}{e(1-\varepsilon)} \tag{25}$$

There is no empirical constant and every parameter has a clear physical meaning. Several parameters have to be determined in this equation, which are the pore area fraction D_f, the maximum pore size $p_{\max}$, the minimum pore size $p_{\min}$, and the tortuosity fractal dimension D_t.

2.2.4 Parameter Determination

2.2.4.1 D_f Determination

The pore area fractal dimension D_f is a parameter for characterizing the complex structure of porous media. It can be obtained based on the box-counting method [8, 9].

2.2.4.2 D_t Determination

The determination of tortuosity fractal dimension can be classified into two categories. One is the box-counting method, which proved successful in ref [22, 23]. The other is the analytical method, and several researchers have applied the method [24,25]. Since it is complicated to obtain the cross-section areas, being parallel to the air flow of the fibrous assemblies, the second method is applied.

As a matter of fact, Eq. 7 can be rewritten as:

$$\tau = \frac{L(p)}{L_0} = \left(\frac{L_0}{p}\right)^{D_t - 1} \tag{26}$$

The average tortuosity (τ_m) can be determined by substituting the average pore diameter (p_m) into Eq. 21.

$$\tau_m = \left(\frac{L_0}{p_m}\right)^{D_t - 1} \tag{27}$$

Then, the fractal dimension, D_t, for tortuous flow stream tubes in porous media can be obtained from Eq. 21 as:

$$D_t = 1 + \frac{\ln \tau_m}{\ln \dfrac{L_0}{p_m}} \tag{28}$$

In the above equation, the average pore diameter p_m can be determined by Eq. 29.

$$p_m = \int_{p_{\min}}^{p_{\max}} p f(p) dp = \int_{p_{\min}}^{p_{\max}} p D_f p_{\min}^{D_f} p^{-(1+D_f)} dp = \frac{D_f}{D_f - 1} p_{\min} \left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_f - 1}\right] \tag{29}$$

The tortuosity model of fibrous assemblies developed by Koponen [26] is selected for calculation here, as it was proved to be more accurate.

$$\tau = 1 + 0.65 \frac{(1 - \varepsilon)}{(\varepsilon - 0.33)^{0.19}} \tag{30}$$

Accordingly, the tortuosity fractal dimension can be derived by substituting Eq. 23 and 24 into Eq. 22.

$$D_t = 1 + \frac{\ln \tau_m}{\ln \dfrac{L_0}{p_m}} = 1 + \frac{\ln \left[1 + 0.65 \dfrac{(1 - \varepsilon)}{(\varepsilon - 0.33)^{0.19}}\right]}{\ln \dfrac{L_0 (D_f - 1)}{D_f p_{\min} \left[1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{D_f - 1}\right]}} \tag{31}$$

2.2.4.3 $p_{\max}$, $p_{\min}$ Determination

The largest pore size $p_{\max}$ in fibrous assemblies has been discussed in a series of papers [27, 28]. The expression can be [28]:

$$p_{\max} = \frac{2.459}{\omega} \tag{32}$$

$$\omega = \frac{4\mu}{\pi L_0 R' \rho_f} \tag{33}$$

Where ω is the total length of fibers per unit area, μ is the mass per unit area, and ρ_f is the density of fiber.

The minimum pore size $p_{\min}$ in fibrous assemblies has not been found in previous studies, and it is assumed to be expressed in Eq. 34.

$$p_{\min} = \frac{\theta}{\omega} \tag{34}$$

Where θ is the constant determined by comparing the experimental and theoretical results.

3 Results and Discussion

3.1 Experimental

A variety of nonwoven fabrics were selected for the samples. The specifications of these samples are listed in Table 1. The thickness was measured in accordance with ISO 5084-1996. The pressing pressure is 100 cN and the pressing time is 10 s. The area density was calculated by measuring the weight of a sample with the diameter of 10 cm using an electronical balance with the precision of 0.00001 g. The porosity of the samples is calculated by the following equation:

$$\varepsilon = 1 - \frac{\mu}{AL_0\rho_f} \tag{35}$$

Table 1: Parameters of the nonwoven fabrics chosen

Sample number	Fiber type	Thickness (mm)	Area density (g/m^2)	Porosity (%)
1	PET-100%	0.449	34	0.945
2	PET/VS-30/70	0.49	42.46	0.941
3	PET/VS-70/30	0.633	65.7	0.927
4	PET/VS-70/30	0.817	84.5	0.927
5	PP	0.317	44.5	0.845
6	PP	0.464	67.4	0.840
8	PP	0.63	101.3	0.809
10	PET-100%	0.615	141	0.846
11	Basalt	3.25	581	0.936

The effective heat conductivity was measured by the Kawabata Thermolabo, which is in accordance to the Chinese National Standard GB11048-89. The temperature difference ΔT between the two sides of a sample is fixed during the testing, and is set to be 10℃ here. The pressure applied to the sample is controlled to be 6 g/cm^2 here. According to Fourier law, the heat conductivity k_{exp} can be expressed as:

$$k_{\mathrm{exp}} = \frac{qL_0}{A\Delta T} \tag{36}$$

3.2 The Comparison of the Effective Heat Conductivities Between Fractal and Experimental Results

It can be seen in Fig. 1 that the effective heat conductivities by theoretical method and experiment are in good accordance with each other, indicating the validation of the theoretical method.

The parameters m and θ are determined to be 0.5 and 0.1 correspondingly. The parameter m determined here is similar to the value got by Liu in his model [29]. The minimum pore diameters of samples 2, 4, and 11 are calculated to be 7.5 μm, 8.4 μm, and 5.5 μm, very close to the results of the experiment done by Yang [30], which are 7.37 μm, 8.27 μm, and 5.32 μm.

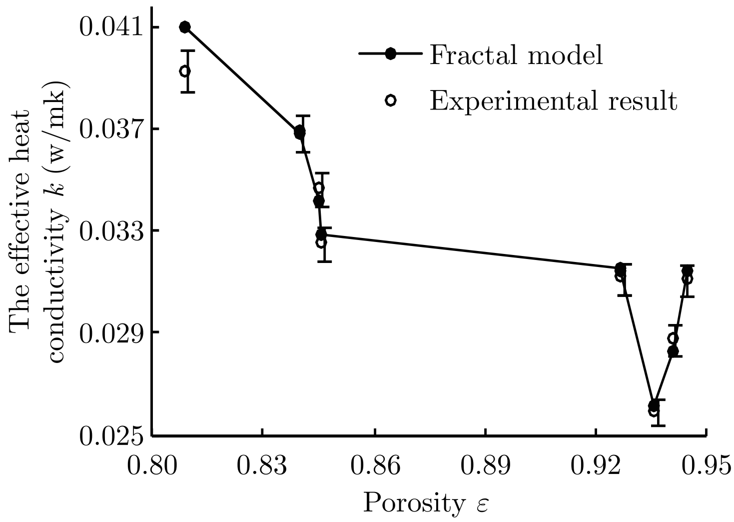

Fig. 1: The effective heat conductivity versus porosity by theoretical method and experiment

4 Conclusion

In this paper, a fractal model was developed for studying the effective heat conductivity of fibrous assemblies. The fractal model indicates that the effective heat conductivity is related to the parameters, including the pore area fractal dimension, tortuosity fractal dimension, the maximum and minimum pore diameters, the solid conductivity, air conductivity, porosity and the ratio of the number of perpendicular channels to the total number of channels.

The theoretical results were compared with the experimental results, and good accordance was obtained.

The parameter m is determined to be 0.5, indicating the fiber contact influence on solid heat conductivity. The constant θ in the minimum pore size equation is determined to be 0.1.

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