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General Interpolation Formulae for Barycentric Blending Interpolation

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Abstract. General interpolation formulae for barycentric interpolation and barycentric rational Hermite interpolation are established by introducing multiple parameters, which include many kinds of barycentric interpolation and barycentric rational Hermite interpolation. We discussed the interpolation theorem, dual interpolation and special cases. Numerical example is given to show the effectiveness of the method.

Key Words: General interpolation formulae of interpolation, barycentric interpolation, barycentric rational Hermite interpolation.

AMS Subject Classifications: 41A20, 65D05

1 Introduction

Developing numerical methods for computing approximations of analytic functions by means of polynomials and rational functions represents a fundamental research area of computational mathematics. Lagrangian interpolation, Newton interpolation and Thiele-type continued fractions interpolation may be the favoured linear interpolation and non-linear interpolation. Lagrangian interpolation is praised for analytic utility and beauty but deplored for numerical practice [1]. The advantages of barycentric interpolation formulations in computation are small number of floating point operations (flops) and good numerical stability. Adding a new data pair, the barycentric interpolation formula don't require renew computation all of basis functions [1, 2]. It can avoid the oscillation of Lagrange interpolation by using barycentric interpolation formulations and second kind of Chebyshev points as interpolating points. In barycentric interpolation formulations, the different weight corresponds to different type of interpolation. The most of these interpolation are barycentric rational interpolation. The barycentric rational interpolations have more advantages than the polynomial interpolation and continued fractions interpolation in computation, for example, easy calculation, the information concerning the existence and location of poles of the interpolation, detection of the unattainable

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points, good numerical stability, the usage of shape control [3]. In the last years several researchers have focused their attention on this subject. For example Berrut and Henrici studied barycentric formulas for trigonometric polynomials, barycentric rational formulas [1, 2, 4–6]. Kahng showed the generalizations of univariate Newton's method and applied it to the approximation problems in 1967 [7]. These generalization extended the applicable interpolation functions from polynomials to rational functions, their transformations and some nonlinear functions. Also, these generalizations enabled us to treat the interpolation in a unified manner. Furthermore, Kahng described a class of interpolation functions and showed the explicit method of osculatory interpolation with a function in the class in 1969 [8]. These two functions have many special cases, such as Newton interpolation polynomial, Thiele-type continued fractions interpolation, Hermite interpolation, Salzer-type osculatory interpolation, trigonometric functions interpolations and so on. In 1999, by introducing multiple parameters, Tan and Fang [9] studied several general interpolation formulae for bivariate interpolation which include many classical interpolant schemes, such as bivariate Newton interpolation, Thiele-type branched continued fractions for two variables, Newton-Thiele's blending rational interpolation, Thiele-Newton's blending rational interpolation, and symmetric branched continued fraction discussed by Cuyt and Murphy et al. Tan discussed more general interpolation grids [10]. Recently Tang and Zou [11] have improved and extended the general interpolation formulae studied by Tan and Fang by introducing multiple parameters, so that the new frames can be used to deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur. The general form of block-based bivariate blending rational interpolation with the error estimation is established by introducing two parameters. From the general form, four different block-based interpolations can be obtained. Then an efficient algorithm for computing bivariate lacunary rational interpolation is constructed based on block-based bivariate blending rational interpolation. Tang and Zou [12, 13] construct general structures of one and two variable interpolation function, without depending on the existence of divided differences or inverse differences, and also discusses the block based osculatory interpolation in one variable case, generalize the conclusion of Kahng to bivariate case.

Our contribution in this paper is to obtain general interpolation formulae for barycentric interpolation by introducing multiple parameters, which include Thiele barycentric blending rational interpolation, Newton barycentric blending rational interpolation, associated continued fractions barycentric blending rational interpolation and their dual schemes, bivariate barycentric interpolation, barycentric Thiele blending rational interpolation, barycentric Newton blending rational interpolation, barycentric associated continued fractions blending rational interpolation, barycentric Hermite blending rational interpolation, barycentric Hermite blending rational interpolation based on Padé approximations and so on as its special cases.

The organization of the paper is as follows. In Section 2 we discuss the general interpolation formulae for barycentric blending interpolation and the dual general interpolation formulae and its special cases. In Section 3, we present general interpolation formulae for barycentric rational Hermite interpolation and its special cases. Numerical example is given to show the effectiveness of the results in Section 4.

2 General interpolation formulae for barycentric blending interpolation

2.1 The construct of general interpolation formulae for barycentric blending interpolation

S. H. Kahng has employed the interpolation function

$$Q(x)=f_0(a_0+g_0(x)f_1(a_1+g_1(x)f_2(a_2+\cdots+g_{n-1}(x)f_n(a_n)\cdots))) \tag{2.1}$$

to treat the univariate interpolation in a unified manner [8]. This function can also be expressed as $Q(x)=f_0\{D_0(x)\}$, where $D_i(x)=a_i+g_j(x)f_{i+1}\{D_{i+1}(x)\}$, $i=0,1,\cdots,n-1$; $j=0,1,\cdots,n-1$, and $D_n(x)=a_n$.

Notations:

$$\begin{aligned}&h(A)=\{h(x)|x\in A\},\\&R(h):\ \text{range of } h(x).\end{aligned}$$

Lemma 2.1 (see [8]). *Given a function $y(x)$ continuous in a finite interval $[a,b]$ and $n+1$ points x_i such that $a\le x_0<x_1<\cdots<x_n\le b$. Then there exists a unique set of parameters $a_0,a_1,\cdots,a_n$ for the interpolation function*

$$Q(x)=f_0(a_0+g_0(x)f_1(a_1+\cdots+g_{n-1}(x)f_n(a_n)\cdots)) \tag{2.2}$$

satisfying $Q(x_i)=y(x_i)$, $i=0,1\cdots,n$, and $Q(x)$ is continuous if

a) f_i is continuous, strictly monotone in $(-\infty,+\infty)$, and the range of $f_i(x)$ covers $(-\infty,+\infty)$, $i=1,2,\cdots,n$,

b) f_0 is continuous and its inverse function f_0^{-1} exists in $R(f_0)$, and $R(f_0)\supset y([a,b])$,

c) functions $g_j(x)$, $j=0,1,\cdots,n-1$ are continuous in $[a,b]$, and

$$g_j(x)\begin{cases}=0, & x=x_j,\\ \neq 0, & x>x_j.\end{cases} \tag{2.3}$$

For simplicity and without lose of generality, we restrict ourselves to the case where bivariate problems are involved, and we only consider the rectangular grid which satisfies the inclusion property, which means that is given a set of two dimension points in R^2, if a point belongs to $\Pi_{n,m}$, then the rectangular subset of points emanate from the origin with the given point as its furthermost corner, and it also lies in $\Pi_{n,m}$. One can consider other grids similarly [10].

Given a set of real points

$$\Pi_{n,m}=\{(x_i,y_j)|i=0,1,\cdots,n;\ j=0,1,\cdots,m\}\subset[a,b]\times[c,d]\subset R^2.$$

Given a bivariate function $f(x,y)$ defined in a domain $[a,b]\times[c,d]$.

Now we construct a function

$$\begin{aligned}Q(x,y)=&S_0(x,y)f_0(A_0(x,y)+S_1(x,y)f_1(A_1(x,y)+S_2(x,y)f_2(A_2(x,y)\\&+\cdots+S_n(x,y)f_n(A_n(x,y))\cdots))),\end{aligned}\tag{2.4}$$

here we choose $A_i(x,y)$, $(i=1,2,\cdots,n)$ as follows

$$A_i(x,y)=g_0(x,y)a_{i,0}+g_1(x,y)a_{i,1}+\cdots+g_m(x,y)a_{i,m}.\tag{2.5}$$

Then we can get a general interpolation formula for barycentric interpolation.

Theorem 2.1. *Given a function $f(x,y)$ continuous in $[a,b]\times[c,d]$ and $(n+1)\times(m+1)$ points (x_i,y_j) such that*

$$a\le x_0<x_1<\cdots<x_n\le b;\quad c\le y_0<y_1<\cdots<y_m\le d.\tag{2.6}$$

If

1) f_i $(i=1,2,\cdots,n)$ are continuous, strictly monotone in their domain and their ranges are $(-\infty,+\infty)$, $i=1,2,\cdots,n$,

2) f_0 is continuous, and its inverse function f_0^{-1} exists in $S(f_0)$ and $S(f_0)\supset f([a,b],y_0)$,

3) the functions $S_0(x,y)=1$, $S_i(x,y)=x-x_{i-1}$, $i=1,\cdots,n$,

$$g_j(x,y)=\frac{u_k}{y-y_k}\Big(\sum_{k=0}^{m}\frac{u_k}{y-y_k}\Big)^{-1},\quad j=0,1,\cdots,m,\tag{2.7}$$

where u_k are barycentric weights,

4) $F^{(\delta_0)}[x_i,y_j]=f(x_i,y_j)=f_{i,j}$, $i=0,1,\cdots,n$; $j=0,1,\cdots,m$,

$$\begin{aligned}a_{i,j}=&F^{(\delta_i)}[x_0,\cdots x_i;y_j]\\=&\Big(\frac{F^{(\delta_{i-1})}[x_0,\cdots x_{i-2},x_i;y_j]-F^{(\delta_{i-1})}[x_0,\cdots x_{i-2},x_{i-1};y_j]}{x_i-x_{i-1}}\Big)^{\delta_i},\end{aligned}\tag{2.8}$$

where $|\delta_i|=1$, suppose all the $a_{i,j}$ exist, then interpolation function

$$\begin{aligned}Q(x,y)=&S_0(x,y)f_0(A_0(x,y)+S_1(x,y)f_1(A_1(x,y)\\&+S_2(x,y)f_2(A_2(x,y)+\cdots+S_n(x,y)f_n(A_n(x,y))\cdots))),\end{aligned}\tag{2.9}$$

where

$$A_i(x,y)=g_0(x,y)a_{i,0}+g_1(x,y)a_{i,1}+\cdots+g_m(x,y)a_{i,m},\quad (i=1,2,\cdots,n),\tag{2.10}$$

satisfies

$$Q(x_i,y_k)=f(x_i,y_k),\quad (i=0,1,\cdots,n;\quad k=0,1,\cdots,m).\tag{2.11}$$

Proof. For $\forall(x_i;y_k)\in\prod_{n,m}$, we can get

$$\begin{aligned}A_i(x_i,y_k)&=g_0(x_i,y_k)a_{i,0}+g_1(x_i,y_k)a_{i,1}+\cdots+g_m(x_i,y_k)a_{i,m}\\&=\sum_{j=0}^{m}\frac{u_j}{y_k-y_j}a_{i,k}\Big(\sum_{j=0}^{m}\frac{u_j}{y_k-y_j}\Big)^{-1}=a_{i,k},\end{aligned}$$

then we have

$$\begin{aligned}Q(x_i,y_k)=&S_0(x_i,y_k)f_0(A_0(x_i,y_k)+S_1(x_i,y_k)f_1(A_1(x_i,y_k)\\&+S_2(x_i,y_k)f_2(A_2(x_i,y_k)+\cdots+S_i(x_i,y_k)f_i(A_i(x_i,y_k))\cdots)))\\=&S_0(x_i,y_k)f_0(a_{0,k}+S_1(x_i,y_k)f_1(a_{1,k}+S_2(x_i,y_k)f_2(a_{2,k}+\cdots+S_i(x_i,y_k)f_i(a_{i,k})\cdots))).\end{aligned}$$

From Lemma 2.1, we can get

$$\begin{aligned}Q(x_i,y_k)=&S_0(x_i,y_k)f_0(a_{0,k}+S_1(x_i,y_k)f_1(a_{1,k}+S_2(x_i,y_k)f_2(a_{2,k}+\cdots\\&+S_i(x_i,y_k)f_i(a_{i,jk})\cdots)))=f(x_i,y_k).\end{aligned}$$

Thus the theorem is proved. ☐

We can modify the frame as follows and get a new formula of barycentric interpolation

$$\begin{aligned}Q(x,y)=&S_0(x,y)A_0(x,y)+S_1(x,y)A_1(x,y)+S_2(x,y)A_2(x,y)+\cdots\\&+S_N(x,y)A_N(x,y), \qquad (2.12a)\\A_i(x,y)=&g_0(x,y)f(a_{i,0}+g_1(x,y)f_1(a_{i,1}+\cdots+g_m(x,y)f_m(a_{i,m})\cdots)). \qquad (2.12b)\end{aligned}$$

We can get the Theorem 2.2 similarly.

Theorem 2.2. *Given a function $f(x,y)$ continuous in $[a,b]\times[c,d]$ and $(n+1)\times(m+1)$ points (x_i,y_j) such that*

$$a\le x_0<x_1<\cdots<x_n\le b;\quad c\le y_0<y_1<\cdots<y_m\le d, \qquad (2.13)$$

If

1) f_i $(i=1,2,\cdots,n)$, are continuous, strictly monotone in their domain and their ranges are $(-\infty,+\infty)$, $i=1,2,\cdots,n$; $j=0,1,\cdots,m$.

2) f_0 is continuous, and its inverse function f_0^{-1} exists in $S(f_0)$ and $S(f_0)\supset f([a,b],y_0)$,

3) the functions $g_0(x,y)=1$, $g_j(x,y)=y-y_{j-1}$, $j=1,\cdots,m$,

$$S_i(x,y)=\frac{u_l}{x-x_l}\Big(\sum_{l=0}^{n}\frac{u_l}{x-x_l}\Big)^{-1},\quad i=0,1,\cdots,n, \qquad (2.14)$$

where u_l is barycentric weight $i=0,1,\cdots,n$, *4)* $F^{(\eta_0)}[x_i,y_j]=f(x_i,y_j)=f_{i,j}$, $i=0,1,\cdots,n$; $j=0,1,\cdots,m$,

$$\begin{aligned} b_{i,j}=&F^{(\eta_j)}[x_i;y_0,\cdots,y_j]\\ =&\Big(\frac{F^{(\eta_{j-1})}[x_i;y_0,\cdots,y_{j-2},y_j]-F^{(\eta_{j-1})}[x_i;y_0,\cdots,y_{j-2},y_{j-1}]}{y_j-y_{j-1}}\Big)^{\eta_j}, \end{aligned} \tag{2.15}$$

where $|\eta_i|=1$, *suppose all the* $b_{i,j}$ *exist, then interpolation function*

$$\begin{aligned} Q(x,y)=&S_0(x,y)A_0(x,y)+S_1(x,y)A_1(x,y)+S_2(x,y)A_2(x,y)\\ &+\cdots+S_n(x,y)A_n(x,y), \end{aligned} \tag{2.16}$$

satisfies

$$Q(x_i,y_j)=f(x_i,y_j),\quad i=0,1,\cdots,n;\quad j=0,1,\cdots,m. \tag{2.17}$$

2.2 Special case

Some of the special cases of the above bivariate interpolation functions defined by formula (2.4) and (2.5) are shown as below:

1) If $f_i(x)=x$, $a_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial divided differences

$$\varphi[x_i;y_j]=f(x_i,y_j), \tag{2.18a}$$

$$\varphi[x_i,x_k;y_j]=\frac{\varphi[x_k;y_j]-\varphi[x_i;y_j]}{x_k-x_i}, \tag{2.18b}$$

$$\varphi[x_i,\cdots,x_r,x_s,x_t;y_j]=\frac{\varphi[x_i,\cdots,x_r,x_t;y_j]-\varphi[x_i,\cdots,x_r,x_s;y_j]}{x_t-x_s}, \tag{2.18c}$$

$$a_{i,j}=\varphi[x_0,x_1,\cdots,x_i;y_j],\quad (i=0,1,\cdots,n;\quad j=0,1,\cdots,m), \tag{2.18d}$$

then $Q(x,y)$ is the barycentric Newton blending rational interpolation [13,14].

2) If $f_0(x)=x$, $f_i(x)=\frac{1}{x}$, $a_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial inverse differences,

$$\psi[x_i;y_j]=f(x_i,y_j), \tag{2.19a}$$

$$\psi[x_i,x_k;y_j]=\frac{x_k-x_i}{\psi[x_k;y_j]-\psi[x_i;y_j]}, \tag{2.19b}$$

$$\psi[x_t,\cdots x_r,x_s,x_i;y_j]=\frac{x_i-x_s}{\psi[x_t,\cdots,x_r,x_i;y_j]-\psi[x_t,\cdots,x_r,x_s;y_j]}, \tag{2.19c}$$

$$a_{i,j}=\psi[x_0,x_1,\cdots x_i;y_j],\quad (i=0,1,\cdots,n;\quad j=0,1,\cdots,m), \tag{2.19d}$$

then $Q(x,y)$ is the barycentric Thiele blending rational interpolation [14].

3) If $f_0(x)=x$, $f_i(x)=x^{(-1)^{i+1}}$, $a_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial divided differences, partial inverse differences

$$\tau[x_i;y_j]=f(x_i,y_j),\quad i=0,1,\cdots,2[n/2]+1;\quad j=0,1,\cdots,2[m/2]+1, \tag{2.20a}$$

$$\tau[x_0,x_1,\cdots,x_{2i-1},x_{2i},x_{2i+1};y_j]=\frac{\tau[x_0,x_1,\cdots,x_{2i-1},x_{2i+1};y_j]-\tau[x_0,x_1,\cdots,x_{2i-1},x_{2i};y_j]}{x_{2i+1}-x_{2i}}, \tag{2.20b}$$

$$\tau[x_0,x_1,\cdots x_{2i},x_{2i+1},x_{2i+2};y_j]=\frac{x_{2i+2}-x_{2i+1}}{\tau[x_0,x_1,\cdots x_{2i},x_{2i+2};y_j]-\tau[x_0,x_1,\cdots x_{2i},x_{2i+1};y_j]}, \tag{2.20c}$$

$$a_{i,j}=\tau[x_0,x_1,\cdots x_i,;y_j],\quad (i=0,1,\cdots,2[n/2]+1;\quad j=0,1,\cdots,2[m/2]+1), \tag{2.20d}$$

then $Q(x,y)$ is the barycentric associated continued fractions blending rational interpolation [13].

4) If $f_0(x)=x$, $f_i(x)=0$, $a_{i,0}=f(x_i)$, $Q(x,y)$ is the univariate barycentric rational interpolation [1,3,15].

5) If $Q(x,y)$ is as showed in (2.4), $A_i(x,y)$ $(i=0,1,\cdots,n)$ is as showed in (2.5), and we choose

$$S_i(x,y)=\frac{u_i}{x-x_i}\Big(\sum_{i=0}^{n}\frac{u_i}{x-x_i}\Big)^{-1},\qquad i=0,1,\cdots,n, \tag{2.21a}$$

$$g_j(x)=\frac{v_j}{y-y_j}\Big(\sum_{j=0}^{m}\frac{v_j}{y-y_j}\Big)^{-1},\qquad j=0,1,\cdots,m, \tag{2.21b}$$

$a_{i,j}(x,y)=a_{i,j}=f(x_i,y_j)$, u_j, v_i $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are barycentric weights, then $Q(x,y)$ is the bivariate barycentric rational interpolation [16].

If we choose the formula (2.12a) are $Q(x,y)$ and (2.12b) are $A_i(x,y)$ $(i=0,1,\cdots,n)$), then the general frame for barycentric blending interpolation has some special cases as shown below:

1) If $f_0(x)=x$, $f_i(x)=\frac{1}{x}$, $b_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial inverse differences

$$\varphi[x_i;y_j]=f(x_i,y_j), \tag{2.22a}$$

$$\varphi[x_i;y_j,y_k]=\frac{y_k-y_j}{\varphi[x_i;y_k]-\varphi[x_i;y_j]}, \tag{2.22b}$$

$$\varphi[x_i;y_j,\cdots y_r,y_s,y_t]=\frac{y_t-y_s}{\varphi[x_i;y_j,\cdots,y_r,y_t]-\varphi[x_i;y_j,\cdots,y_r,y_s]}, \tag{2.22c}$$

$$b_{i,j}=\varphi[x_i;y_0,y_1,\cdots y_j],\quad (i=0,1,\cdots,n;\quad j=0,1,\cdots,m), \tag{2.22d}$$

then $Q(x,y)$ is the Thiele barycentric blending rational interpolation [17].

2) If $f_i(x)=x$, $b_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial divided differences

$$\psi[x_i;y_j]=f(x_i,y_j), \tag{2.23a}$$

$$\psi[x_i;y_j,y_k]=\frac{\psi[x_i;y_k]-\psi[x_i;y_j]}{y_k-y_j}, \tag{2.23b}$$

$$\psi[x_i;y_j,\cdots y_r,y_s,y_t]=\frac{\psi[x_i;y_j,\cdots,y_r,y_t]-\psi[x_i;y_j,\cdots,y_r,y_s]}{y_t-y_s}, \tag{2.23c}$$

$$b_{i,j}=\psi[x_i;y_0,y_1,\cdots y_j],\quad (i=0,1,\cdots,n;\quad j=0,1,\cdots,m), \tag{2.23d}$$

then $Q(x,y)$ is the Newton barycentric blending rational interpolation [14].

3) If $f_0(x)=x$, $f_i(x)=x^{(-1)^{i+1}}$, $b_{i,j}$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$ are partial divided differences, partial inverse differences,

$$\tau[x_i;y_j]=f(x_i,y_j),\quad i=0,1,\cdots,2[n/2]+1;\quad j=0,1,\cdots,2[m/2]+1, \tag{2.24a}$$

$$\tau[x_i;y_0,y_1,\cdots y_{2j-1},y_{2j},y_{2j+1}]=\frac{\tau[x_i;y_0,y_1,\cdots y_{2j-1},y_{2j+1}]-\tau[x_i;y_0,y_1,\cdots y_{2j-1},y_{2j}]}{y_{2j+1}-y_{2j}}, \tag{2.24b}$$

$$\tau[x_i;y_0,y_1,\cdots y_{2j},y_{2j+1},y_{2j+2}]=\frac{y_{2j+2}-y_{2j+1}}{\tau[x_i;y_0,y_1,\cdots y_{2j},y_{2j+2}]-\tau[x_i;y_0,y_1,\cdots y_{2j},y_{2j+1}]}, \tag{2.24c}$$

$$b_{i,j}=\tau[x_i;y_0,y_1,\cdots y_j],\quad (i=0,1,\cdots,2[n/2]+1), \tag{2.24d}$$

then $Q(x,y)$ is the associated continued fractions barycentric blending rational interpolation [18].

4) $f_0(x)=x$, $f_i(x)=0$, $b_{i,j}=f(x_i)$ $(i=0,1,\cdots,n,\ j=0,1,\cdots,m)$, then $Q(x,y)$ is the univariate barycentric rational interpolation [13,15].

Furthermore, one can get more blending rational interpolations via choosing $f_i(x)$ appropriately.

We can see the Theorem 2.1 is obtained starting from the independent variable y, Theorem 2.2 is obtained starting from the independent variable x, of course, one can also obtaine starting from the independent variable x in Theorem 2.1 and variable y in Theorem 2.2. Exchanging the roles of the variables x and y, one can also construct the dual general interpolation formulae.

3 General interpolation formulae of barycentric rational Hermite interpolation

3.1 The construct of general interpolation formulae of barycentric rational Hermite interpolation

Let $R(x)\in R_{m,n}(x)$, where $R_{m,n}(x)$ is the set of all rational functions with the degrees of numerator at most m and the degrees of denominator at most n. The barycentric rational

Hermite interpolation is presented as

$$R(x)=\sum_{i=0}^{n}\sum_{k=0}^{s_i-1}\frac{\omega_{ik}}{(x-x_i)^{k+1}}\sum_{j=0}^{k}\frac{f_i^{(j)}}{j!}(x-x_i)^j\Big(\sum_{i=0}^{n}\sum_{k=0}^{s_i-1}\frac{\omega_{ik}}{(x-x_i)^{k+1}}\Big)^{-1}, \tag{3.1}$$

for the data $\{x_i,f_i^{(j)}\}$, $j=0,1,2,\cdots,s_i-1$, $i=0,1,\cdots,n$, when $i\neq j$, $x_i\neq x_j$. We can construct the general interpolation formulae of barycentric rational Hermite interpolation as

$$R(x)=\sum_{i=0}^{n}\sum_{k=0}^{s_i-1}\frac{\omega_{i,k}}{(x-x_i)^{k+1}}P_{i,k}(x)\sum_{i=0}^{n}\Big(\sum_{k=0}^{s_i-1}\frac{\omega_{i,k}}{(x-x_i)^{k+1}}\Big)^{-1}. \tag{3.2}$$

Theorem 3.1. *The interpolation function* (3.1) *satisfis the interpolaiton conditions*

$$R(x_i)=f_i,\quad R^{(j)}(x_i)=f_i^j,\quad j=1,2,\cdots,s_i-1;\quad i=0,1,\cdots,n. \tag{3.3}$$

One can proof the theorem similar to the method in paper [20,21].

3.2 Special cases

The general interpolation formulae of barycentric rational Hermite interpolation include the following interesting special cases.

Case 1. If we choose $P_k(x)$ as the Newton interpolation polynomial with the same points,

$$P_{i,k}(x)=c_{i,0}+c_{i,1}(x-x_i)+c_{i,2}(x-x_i)^2+\cdots+c_{i,k}(x-x_i)^k, \tag{3.4}$$

then $R(x)$ is a barycentric rational Hermite interpolation [19].

Case 2. Given a function $f(x)$ with given single, former power series of the function $f(x)$ at set $x=x_k$ as shows

$$f(x)=\sum_{i=0}^{\infty}c_i^{(k)}(x-x_k),\quad c_0^{(k)}\neq 0,\quad (k=0,1,\cdots,n). \tag{3.5}$$

We can get an $(s_i-1,1)$ times Padé-type approximation at point $x=x_k$, which is a generator polynomial with $V(x)=v-x$, $v\notin[x_0,x_N]$ (v is a constant)

$$P_{i,k}(x)=\Big(\frac{s_i-1}{1}\Big)_f(x)=\sum_{i=0}^{s_i-1}\frac{a_i(x-x_k)^i}{V(x)}, \tag{3.6}$$

where a_i is the coefficient of the Taylor series expansion of $f(x)V(x)$ at point $x=x_k$. Then $R(x)$ is a composite barycentric rational Hermite interpolation based on the Padé-type approximation [20].

Case 3. If $P_{i,k}(x)$ is the Salzer-type osculatory rational interpolation

$$P_{i,k}(x)=d_{i,0}+\frac{x-x_i|}{|\, d_{i,1}}+\frac{x-x_i|}{|\, d_{i,2}}+\cdots+\frac{x-x_i|}{|\, d_{i,k}}, \tag{3.7}$$

then $R(x)$ is a new type composite barycentric rational Hermite interpolation [21].

One can choose appropriate barycentric weight to deal with the interpolation problems where unattainable points may occur. We can also choose the Padé approximation or perturbed Padé-type approximation as $P_{i,k}(x)$. We can choose the nodes to simplify the barycentric weights, for example the Chebyshev nodes and so on. If the Chebyshev series of the given function is given, one can construct the Chebyshev-Padé approximation, Chebyshev-Padé-type Approximation and perturbation Chebyshev-Padé approximation, and then one can construct some new forms of the composite Hermite blending rational interpolation scheme.

4 Numerical example

In this section, we take simple example to show the effectiveness of the results in this paper.

Suppose the function values $f(x_i,y_j)$ of

$$f(x,y)=\frac{x}{1+x^2}+\frac{y}{1+y^2}$$

are given as follows:

Table 1: Interpolation data.

	$y_0=0$	$y_1=1$	$y_2=2$
$x_0=0$	0	0.5	0.4
$x_1=1$	0.5	1	0.9
$x_2=2$	0.4	0.9	0.8

Using the frame in the paper, one can get many special interpolations, some of them are as follows:

Scheme 1: Thiele barycentric blending rational interpolation

$$Q_2(x,y)=\frac{\frac{1}{x}\left(\frac{y}{2+\frac{y-1}{-\frac{10}{3}}}\right)+\frac{-1}{x-1}\left(\frac{1}{2}+\frac{y}{2+\frac{y-1}{-\frac{10}{3}}}\right)+\frac{1}{x-2}\left(\frac{2}{5}+\frac{y}{2+\frac{y-1}{-\frac{10}{3}}}\right)}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}},$$

Scheme 2: Associated continued fractions barycentric blending rational interpolation

$$Q_3(x,y)=\frac{\frac{1}{x}\left(\frac{y}{2}+\frac{y(y-1)}{-\frac{10}{3}}\right)+\frac{-1}{x-1}\left(\frac{1}{2}+\frac{y}{2}+\frac{y(y-1)}{-\frac{10}{3}}\right)+\frac{1}{x-2}\left(\frac{2}{5}+\frac{y}{2}+\frac{y(y-1)}{-\frac{10}{3}}\right)}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}$$
$$=\frac{-3x^2y^2+8x^2y+6xy^2-x^2-6y^2-16xy+6x+16y}{10x^2-20x+20},$$

Scheme 3: Bivariate barycentric blending rational interpolation

$$Q_4(x,y)=\frac{\frac{1}{y}\left(\frac{\frac{1}{x}(0)+\frac{-1}{x-1}(\frac{1}{2})+\frac{1}{x-2}(\frac{1}{2})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}\right)+\frac{-1}{y-1}\left(\frac{\frac{1}{x}(\frac{1}{2})+\frac{-1}{x-1}(1)+\frac{1}{x-2}(\frac{9}{10})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}\right)}{\frac{1}{y}+\frac{-1}{y-1}+\frac{1}{y-2}}$$
$$+\frac{\frac{1}{y-2}\left(\frac{\frac{1}{x}(\frac{2}{5})+\frac{-1}{x-1}(\frac{9}{10})+\frac{1}{x-2}(\frac{4}{5})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}\right)}{\frac{1}{y}+\frac{-1}{y-1}+\frac{1}{y-2}},$$

Scheme 4: Barycentric Newton blending rational interpolation

$$Q_5(x,y)=\frac{\frac{1}{x}(0)+\frac{-1}{x-1}(2)+\frac{1}{x-2}(-\frac{10}{3})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}+\frac{\frac{1}{x}(\frac{1}{2})+\frac{-1}{x-1}(2)+\frac{1}{x-2}(-\frac{10}{3})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}y$$
$$+\frac{\frac{1}{x}(\frac{2}{5})+\frac{-1}{x-1}(2)+\frac{1}{x-2}(-\frac{10}{3})}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}y(y-1)$$
$$=\frac{-3x^2y^2+6x^2y+8xy^2-6x^2-y^2-16xy+16x+6y}{10y^2-20y+20},$$

Scheme 5: Newton barycentric blending rational interpolation

$$Q_6(x,y)=\frac{\frac{1}{x}(\frac{y}{2}-\frac{3}{10}y(y-1))+\frac{-1}{x-1}(\frac{1}{2}+\frac{y}{2}-\frac{3}{10}y(y-1))+\frac{1}{x-2}(\frac{2}{5}+\frac{y}{2}-\frac{3}{10}y(y-1))}{\frac{1}{x}+\frac{-1}{x-1}+\frac{1}{x-2}}$$
$$=\frac{-3x^2y^2+8x^2y+6xy^2-x^2-6y^2-16xy+6x+16y}{10x^2-20x+20}.$$

It is easy to verify

$$Q_s(x_i,y_j)=f(x_i,y_j),\quad i=0,1,2;\quad j=0,1,2,\quad s=1,2,\cdots,5.$$

5 Conclusions

In practical applications, the choice of f_i's may be determined by the desired form of interpolation, e.g., polynomial, rational function of given degree of the numerator and the denominator, or other function schemes. If there is no restriction to the form of $Q(x,y)$, the best choice may be the interpolation function which gives the smallest error term among the functions certain complexity. However, it is difficult to determine such a function without the process of trial and comparison.

General interpolation formulae of barycentric blending interpolation function is constructed, many classical interpolation schemes are its special case, and we can approximate function by choosing barycentric weight. Clearly our methods provide us with many flexible interpolation schemes for choices. Another question is coming, there are so many schemes which we can use, how to choose a formula appropriately is our further work.

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