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REGULAR ARTICLE

The Velocity Gauge KFR Ionization Rates of $\mathbf{H(2p_x)}$, $\mathbf{H(2p_y)}$ and $\mathbf{H(2p_z)}$ Atom in the Linear Polarized Laser Field

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Abstract We obtain the ionization rates of $\mathrm{H}(2p_x)$, $\mathrm{H}(2p_y)$ and $\mathrm{H}(2p_z)$ from the so-called velocity gauge(VG) forms of the Keldysh-Faisal-Reiss(KFR) theory for a linear polarized laser field. Then the ionization rates of $\mathrm{H}(2p_x)$, $\mathrm{H}(2p_y)$ and $\mathrm{H}(2p_z)$ were compared thoroughly. The result of ionization rates of $\mathrm{H}(2p_x)$ and $\mathrm{H}(2p_y)$ are totally equal , while different from that of $\mathrm{H}(2p_z)$. We also numerically compare ionization rates of $\mathrm{H}(2p_x)$, $\mathrm{H}(2p_y)$ and $\mathrm{H}(2p_z)$, the numerical study shows that the ionization rate of $\mathrm{H}(2p_z)$ is a few orders underestimated compared with the ionization rate of $\mathrm{H}(2p_x)$ and H $\mathrm{H}(2p_y)$. Our ionization rates of $\mathrm{H}(2p_x)$, $\mathrm{H}(2p_y)$ and $\mathrm{H}(2p_z)$ may provide more insight into the origin of the discrepancy between the different bound states for hydrogen atom.

AMS subject classifications: 37N20
Keywords: strong-field approximation, gauge invariance, hydrogen atom

1. Introduction

With the rapid advancement of laser technology, the physics of strong-field photoionization has been extensively studied. Much of our knowledge of strong-field photoionization from the study of above-threshold ionization (ATI) [1]. Since then, ATI has been continuously advancing our understanding of strong-field physics. As in atom systems, a series of related strong-field processes occur including above-threshold ionization and dissociation [2,3], double and multiple ionization [4,5], and high-order harmonic generation [6,7]. Especially from the physical point of view using linear polarization of the laser field is more interesting,

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since one can experimentally observe such phenomena like rescattering. The hydrogen atom is the simplest atom and as such has been the favorite of theorists in investigations of atom effects in strong-field physics. To understand the physics of strong-field photoionization, we believe that detailed studies of hydrogen atom ionization rates of different bound states will provide additional insights into the dynamics of ionization processes in general. As we have already investigated on the ionization rate of H(1s) in intense laser field, the ionization rate of H($2p_x$), H($2p_y$) and H($2p_z$) in intense laser field is supposed to be obtained .Therefore, it is intensely important to explore the hydrogen atom photoionization in more detail.

Certainly, in the study of strong-field ionization the two model the velocity gauge(VG) and the length gauge(LG) forms of the Keldysh-Faisal-Reiss(KFR) theory [8-13] should been widely used. The KFR theory is based on the S-matrix theory; approximate wavefunctions have to be used in practice to evaluate the S-matrix elements [14] for bound-free transitions. It utilizes the S-matrix theory, which is in principle exact. However, since there is no general analytical solution to the Schrödinger equation for a charged particle interacting. Processing for numerically integrate the TDSE [15] (the time-dependent Schrödinger equation) is difficult and its approach has limitation. The strong-field approximation (SFA) in the VG and the LG differ and apparently constitute two distinct models. Various approximate theories may lead to different expressions for the ionization rate. All theories describe the same physical problem, and the main difference between them is in the form of the laser-atom interaction. It have caused an extensive controversy about which gauge is more appropriate for the SFA [16-21]. Meanwhile, the gauge invariant is theoretically proved still valid for photoionization amplitude [22-27]. However, which indicates similar or even more pronounced gauge dependencies. For example, within the same approximations both VG and LG KFR theories describe the same electromagnetic fields acting on bound electron, we found the VG and LG KFR [28,29] ionization rates of the H(1s) atom are different. Furthermore, the VG SFA predicts better well about phenomenon. To summarize, numerous controversial results obtained within different gauges and needs some revising and improving.

F.–C. Ma has derived a formula of atom ionization in an intense field using the VG [30,33] which provides a more insightful guidance to investigate the long standing discrepancy in the strong-field ionization theory. The formula is as simple as Keldysh's formula, and he draw a conclusion that the discrepancy of calculated ionization rate between the approximations in two gauges do not arise from algebraic method. Tang Z H also has obtained a formula of molecular ionization in an intense field [33]. The method which he used can date from Ma F C's formula and he changed conditions in VG forms of the KFR theory for a linear polarized laser field. He presented an appropriate example for strong-field ionization of N_2 molecules, which the corresponding approximate two centered single-electron molecular initial bound wave function can be expressed as an appropriate linear combination of hydrogen like $2p_z$ atomic orbitals. Bauer has numerically compared the ionization rates and photoelectron spectra in LG and VG forms of the KFR theory for a circularly polarized laser field [31]. He obtained a conclusion that both forms of the KFR theories show qualitatively different behavior of ionization rate as a function of frequency and intensity of the laser field. In the LG KFR theory, one obtains much smaller stabilization of the hydrogen atom than in the VG KFR theory. In addition, Lin together with his co-workers [32] has generalized and improved the derivation of photoionization rate formula for one-electron atoms proposed by Keldysh to the counterpart of randomly oriented diatomic molecules in LG KFR theory. So in this paper, we have a very good theoretical reason to investigate ionization rate in VG in the linear polarized laser field even without Coulomb corrections in the final state. The purpose of our paper is to deepen existing KFR theory in the VG.

Furthermore, we used the S-matrix formalism of conventional SFA by the standard linear combination of atomic orbitals method utilized for approximate analytical wave function of an initial atom bound state. We use the saddle point approximation to perform the contour integral, and assume that the kinetic momentum of the free electron is small, satisfying the condition $p^2 \ll 2E_b$.

Therefore our interest here is to extend such an analysis for the comparison of the ionization rate in the VG KFR theory and excited states of the hydrogen atom in what follows we use atomic units: $m_e = \hbar = e = 1$, substituting explicitly -1 for the electronic charge.

2. Methods

Our aim is to derive a more exact ionization rate for H($2p_x$), H($2p_y$) and H($2p_z$), which is subject to an intense laser field (linear polarization). In this paper we use the following initial-state wavefunctions [31], which in the spatial representation are (θ', φ' denote the polar and azimuthal angles of $\vec{r}$):

$$\phi_{2p_x}(\vec{r}) = \sqrt{\frac{Z^5}{32\pi}} r \exp(-Zr/2) \sin\theta' \cos\varphi', \tag{1}$$

$$\phi_{2p_y}(\vec{r}) = \sqrt{\frac{Z^5}{32\pi}} r \exp(-Zr/2) \sin\theta' \sin\varphi', \tag{2}$$

$$\phi_{2p_z}\left(\vec{r}\right)=\sqrt{\frac{Z^5}{32\pi}}r\exp\left(-Zr/2\right)\cos\theta', \tag{3}$$

In the momentum representation these wavefunctions are (θ, φ denote the polar and azimuthal angles of $\vec{p}$) respectively

$$\phi_{2p_x}\left(\vec{p}\right)=\int\frac{d^3r}{\left(2\pi\right)^{\frac{3}{2}}}\exp\left(-i\vec{p}\vec{r}\right)\phi_{2p_x}\left(\vec{r}\right)=\frac{i\sqrt{Z^7}}{\pi}\frac{p\sin\theta\cos\varphi}{\left(Z^2/4+p^2\right)^3}, \tag{4}$$

$$\phi_{2p_y}\left(\vec{p}\right)=\int\frac{d^3r}{\left(2\pi\right)^{\frac{3}{2}}}\exp\left(-i\vec{p}\vec{r}\right)\phi_{2p_y}\left(\vec{r}\right)=\frac{i\sqrt{Z^7}}{\pi}\frac{p\sin\theta\sin\varphi}{\left(Z^2/4+p^2\right)^3}, \tag{5}$$

$$\phi_{2p_z}\left(\vec{p}\right)=\int\frac{d^3r}{\left(2\pi\right)^{\frac{3}{2}}}\exp\left(-i\vec{p}\vec{r}\right)\phi_{2p_z}\left(\vec{r}\right)=\frac{-i\sqrt{Z^7}}{\pi}\frac{p\cos\theta}{\left(Z^2/4+p^2\right)^3}, \tag{6}$$

where $Z=\frac{2}{a}, E_b=\frac{1}{2a^2}$, and Z is the effective nuclear charge, E_b is the binding energy of the atom, so

$$\phi_{2p_x}\left(\vec{p}\right)=\frac{8i}{\pi\sqrt{a^7}}\frac{p\sin\theta\cos\varphi}{\left(E_b+\frac{p^2}{2}\right)^3}, \tag{7}$$

$$\phi_{2p_y}\left(\vec{p}\right)=\frac{8i}{\pi\sqrt{a^7}}\frac{p\sin\theta\sin\varphi}{\left(E_b+\frac{p^2}{2}\right)^3}, \tag{8}$$

$$\phi_{2p_z}\left(\vec{p}\right)=\frac{-8i}{\pi\sqrt{a^7}}\frac{p\cos\theta}{\left(E_b+\frac{p^2}{2}\right)^3}. \tag{9}$$

The VG KFR theory is based on the following approximate S-matrix element, where one considers the transition from a field-free initial state to a Volkov final state directly. The S-matrix can be written as [10]

$$\left(S-1\right)_{fi}=-i\int_{-\infty}^{\infty}dt\left(\psi_f, H_A\phi_i\right)_t, \tag{10}$$

where Ψ_f represents the state vector of the final continuum state, i.e. the Volkov state and ϕ_i represents the initial state of the unperturbed system with no field present. H_A is the interaction Hamiltonian, in the VG or in the LG, respectively ($\vec{p}=-i\vec{\nabla}$):

$$H_A(t)=\vec{A}(t)\cdot\left(-i\vec{\nabla}\right)+\frac{1}{2}\vec{A}(t)^2,\quad \vec{A}(t)=A\vec{\varepsilon}\cos\omega t, \tag{11}$$

where it in the dipole approximation the linear polarized laser field. ε represents the unit vector in the direction of polarization, ω is the frequency of the laser field, A is amplitude of the vector potential. $\vec{F}(t)=-\frac{\partial\vec{A}}{\partial t}=F\vec{\varepsilon}\sin\omega t, F=A\omega$, F is amplitude of the electric field.

$$\psi_f=\frac{1}{\sqrt{V}}\exp\left[i\vec{p}\cdot\vec{r}-\frac{i}{2}p^2t-i\int_{-\infty}^{t}d\tau H_A(\vec{p},\tau)\right], \tag{12}$$

and V is a normalization volume.

$$(S-1)_{fi}=\frac{i}{\sqrt{V}}\phi_i(\vec{p})\left(\frac{p^2}{2}+E_b\right)L(\vec{p},t), \tag{13}$$

$$L(\vec{p},t)=\int_{-\infty}^{\infty}\exp\left[iS_p(t)\right]dt, \tag{14}$$

where

$$S_p(t)=\int_{-\infty}^{t}\left[E_b+\frac{1}{2}\left(\vec{p}+\vec{A}\right)^2\right]d\tau. \tag{15}$$

The following VG probability amplitudes of ionization can be obtained

$$(S-1)_{fi}=\frac{i}{\sqrt{V}}\phi_i(\vec{p})\left(\frac{p^2}{2}+E_b\right)\int_{-\infty}^{\infty}dt\exp\left[i\left(\frac{p^2}{2}+E_b+U_p\right)t-i\xi\sin\omega t+i\frac{z'}{2}\sin 2\omega t\right], \tag{16}$$

where $\xi=-\frac{A}{\omega}p\cos\theta,\ z'=\frac{A^2}{4\omega},\ E_b=-E_i,\ U_P=\frac{A^2}{4}=\frac{I}{4\omega^2}$ is the ponderomotive potential, and I stands for the radiation intensity in atomic units(for linear polarization I = F^2,with F being the electric field vector amplitude).

Using the generalized Bessel function:

$$\exp\left[-i\xi\sin x+i\frac{z'}{2}\sin 2x\right]=\sum_{n=-\infty}^{\infty}J_n\left(\xi,-\frac{z'}{2}\right)e^{-inx}. \tag{17}$$

Then,

$$L(\vec{p},t)=\sum_{n=-\infty}^{\infty}J_n\left(\xi,-\frac{z'}{2}\right)2\pi\delta\left(\frac{p^2}{2}+E_b+U_p-n\omega\right) \tag{18}$$

and

$$J_n\left(\xi,-\frac{z'}{2}\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi}dx\exp\left[i\left(nx-\xi\sin x+\frac{z'}{2}\sin 2x\right)\right]. \tag{19}$$

So Eq.(16) can be written like as

$$\left(S-1\right)_{fi}=\frac{i}{\sqrt{V}}\varphi_i\left(\vec{p}\right)\left(\frac{p^2}{2}+E_b\right)\sum_{n=n_0}^{\infty}\int_{-\pi}^{\pi}dx\exp\left[iS_p\left(x\right)\right]\delta\left[\frac{p^2}{2}+E_b+\left(z'-n\right)\omega\right]. \quad (20)$$

The equation (20) which makes the integral over $x=\omega t$ can be calculated using the saddle point method. Where a complex variable $u=cos\,x$, then

$$S_p\left(u\right)=f\left(u\right)\left(n\cos^{-1}u-\xi\sqrt{1-u^2}+z'u\sqrt{1-u^2}\right), \quad (21)$$

where $f(u)=\pm1$, the positions of the saddle points are decided by $\frac{dS_p(u)}{du}=0$. Then

$$2z'u_s^2-\xi u_s+n-z'=0. \quad (22)$$

We get the saddle point

$$u_s=-\gamma\left[\frac{p\cos\theta}{k}\pm i\left(1+\frac{p^2\sin^2\theta}{2k^2}\right)\right], \quad (23)$$

where $s=1,2, k=\sqrt{2E_b}$, the Keldysh parameter is $\gamma=\frac{\omega\sqrt{2E_b}}{F}=\frac{k}{A}$.

Then we obtain

$$L\left(\vec{p},u\right)=\sum_{n=n_0}^{\infty}\oint_c\frac{-du}{f\left(u\right)\sqrt{1-u^2}}\exp\left[iS_p\left(u\right)\right]\delta\left[\frac{p^2}{2}+E_b+\left(z-n\right)\omega\right]. \quad (24)$$

Then we change Eq.(20) to a contour integral in the complex plain

$$\left(S-1\right)_{fi}=\frac{i}{\sqrt{V}}\varphi_i\left(\vec{p}\right)\left(\frac{p^2}{2}+E_b\right)\sum_{n=n_0}^{\infty}\oint_c\frac{-du}{f(u)\sqrt{1-u^2}}\exp\left[iS_p\left(u\right)\right]\delta\left[\frac{p^2}{2}+E_b+\left(z'-n\right)\omega\right]. \quad (25)$$

Using the method similar to Gribakin and Kuchiev [34], we obtain

$$\left(S-1\right)_{fi}=\frac{i}{\sqrt{V}}\varphi_i\left(\vec{p}\right)\left(\frac{p^2}{2}+E_b\right)\sum_{n=n_0}^{\infty}\sum_{s=1}^{2}\frac{-1}{f\left(u\right)\sqrt{1-u_s}}\left[\frac{2\pi\sqrt{1-u_s}}{-i\left(\xi-4zu_s\right)}\right]^{\frac{1}{2}}\exp\left[iS_p\left(u_s\right)\right]$$

$$\times\delta\left[\frac{p^2}{2}+E_b+\left(z'-n\right)\omega\right]. \quad (26)$$

The total transition probability from the atom ground bound state to the Volkov state is

$$W=\lim_{t\to\infty}\frac{V}{t}\frac{1}{\left(2\pi\right)^3}\int d^3\vec{p}\left|\left(S-1\right)_{fi}\right|^2. \quad (27)$$

Then, we obtain

$$W_{2p_x}=\frac{4}{\pi^4\sqrt{E_b}Fa^7\omega^2\sqrt{1+\gamma^2}}S(\gamma,F,E_b)_{2p_x}\exp\left[-\frac{4\sqrt{2}E_b^{3/2}}{3F}\left(1-\frac{\gamma^2}{10}\right)\right], \quad (28)$$

where

$$S(\gamma,F,E_b)_{2p_x}=\frac{1}{\sqrt{\overline{C}}}\sum_{n=n_0}^{\infty}\frac{2\omega\left[n-\left(U_p+E_b\right)/\omega\right]}{\left(n-z'\right)^4}\exp\left[-\overline{B}\left(n-\frac{U_p+E_b}{\omega}\right)\right]\Phi(\mu)_{2p_x} \quad (29)$$

$$\overline{B}=\frac{2\gamma^3}{3}\left(1-B\right), \quad (30)$$

$$\overline{C}=\frac{\sqrt{E_b}}{\sqrt{2}F\sqrt{1+\gamma^2}}\left(1+C\right). \quad (31)$$

Using the notations

$$B=\frac{3F}{4\sqrt{2}E_b^{3/2}\left(1+\gamma^2\right)^2}, \quad (32)$$

$$C=\frac{F\left(1+4\gamma^2\right)}{4\sqrt{2}E_b^{3/2}\left(1+\gamma^2\right)^{3/2}}, \quad (33)$$

$$\Phi(\mu)_{2p_x}=\left(2\psi\left[\mu^{\frac{1}{2}}\right]+\frac{\psi\left[\mu^{\frac{1}{2}}\right]}{\mu}-\frac{1}{\mu^{\frac{1}{2}}}\right), \quad (34)$$

where $\Psi[\mu]$ is the Dawson's integral, with

$$\mu=4\omega\overline{C}\left(n-\frac{U_p+E_b}{\omega}\right) \quad (35)$$

we obtain

$$W_{2p_y}=\frac{4}{\pi^4\sqrt{E_b}Fa^7\omega^2\sqrt{1+\gamma^2}}S(\gamma,F,E_b)_{2p_y}\exp\left[-\frac{4\sqrt{2}E_b^{3/2}}{3F}\left(1-\frac{\gamma^2}{10}\right)\right], \quad (36)$$

$$W_{2p_z}=\frac{8}{\pi^4\sqrt{E_b}Fa^7\omega^2\sqrt{1+\gamma^2}}S(\gamma,F,E_b)_{2p_z}\exp\left[-\frac{4\sqrt{2}E_b^{3/2}}{3F}\left(1-\frac{\gamma^2}{10}\right)\right], \quad (37)$$

where

$$S(\gamma,F,E_b)_{2p_y}=\frac{1}{\sqrt{\overline{C}}}\sum_{n=n_0}^{\infty}\frac{2\omega\left[n-\left(U_p+E_b\right)/\omega\right]}{\left(n-z'\right)^4}\exp\left[-\overline{B}\left(n-\frac{U_p+E_b}{\omega}\right)\right]\Phi(\mu)_{2p_y}, \quad (38)$$

$$S(\gamma,F,E_b)_{2p_z}=\frac{1}{\sqrt{\overline{C}}}\sum_{n=n_0}^{\infty}\frac{2\omega\left[n-\left(U_p+E_b\right)/\omega\right]}{\left(n-z'\right)^4}\exp\left[-\overline{B}\left(n-\frac{U_p+E_b}{\omega}\right)\right]\Phi(\mu)_{2p_z}, \quad (39)$$

$$\Phi(\mu)_{2p_y}=\left(2\psi\left[\mu^{\frac{1}{2}}\right]+\frac{\psi\left[\mu^{\frac{1}{2}}\right]}{\mu}-\frac{1}{\mu^{\frac{1}{2}}}\right), \quad (40)$$

$$\Phi(\mu)_{2p_z}=\left(\mu^{\frac{1}{2}}-\frac{\psi\left[\mu^{\frac{1}{2}}\right]}{\mu}\right). \quad (41)$$

One should expect that the KFR theory improves as the laser field becomes stronger. The following two conditions determine the lower and the upper applicability limits of the KFR theory:

$$z_1\equiv\frac{2U_p}{E_B}\gg 1,\quad z_f\equiv\frac{2U_p}{c^2}\ll 1. \quad (42)$$

Eq. (41) implies that the ponderomotive potential of the outgoing electron should be much greater than the electron binding energy E_B, and much less than the electron rest energy. The latter condition is due to the non-relativistic description of the process. The Keldysh parameter γ is connected with the Reiss parameter z_1 ($z_1=\frac{1}{\gamma^2}$ for linear polarization).

The main result of this paper is the ionization rates of H($2p_x$) (the same is for H($2p_y$)) and H($2p_z$), respectively. We could see the ionization rates of H($2p_x$) (the same is for H($2p_y$)) and H($2p_z$) show similar structures. The different parts are the parameter and the function of $\Phi(\mu)$ which use the Dawson's integral.

3. Numerical Results and discussion

In this section, we numerically present the results [Eq. (28), Eq. (36) and Eq. (37)] for the ionization rates of H($2p_x$), H($2p_y$) and H($2p_z$) exposed to the linearly polarized laser field in the VG KFR theory. The expressions we obtain in this paper [Eq. (28), Eq. (36) and Eq. (37)]

are actual the generalization of Reiss theory. To estimate accurately hydrogen ionization rates and understand the origin of the discrepancy between the ionization rates of H($2p_x$), H($2p_y$) and H($2p_z$), we investigate the ionization rate as a function of the parameter pair (ω, z_1), for which the condition $\omega \ll 1$ and $z_1 \gg 1$ should be satisfied. In numerical calculations of hydrogen ionization rates, we have decided to fix either the Reiss parameter at z_1=100, or the laser frequency at ω =0.01 a.u.. Thereby our parameters lie well within the range described by the two conditions $\omega \ll 1$ a.u. and $z_1 \gg 1$. In figures 1-3 (where both theories give identical results) the ionization rate of H($2p_x$) (the same is for the ionization rate of H($2p_y$)) are shown by red lines and the ionization rate of H($2p_z$) by green line. There are three ionization rates plotted in **Figure 1** as a function of the laser frequency ω for z_1 =100. Each curve in **Figure 1** begins when z_1 =100 (we have numerically verified that, when the frequency ω tends to zero, each ionization rate shown here also tends to zero). The ionization rate for very high frequency becomes a increasing function of frequency. The ionization rate of H($2p_x$) is usually many orders of magnitude greater than the ionization rate of H($2p_z$). The most striking difference between the ionization rate of H($2p_x$) and H($2p_z$) in **Figure 1** lies in the fact that only for the latter one ionization rate becomes dependent of the laser frequency for sufficiently strong laser field.

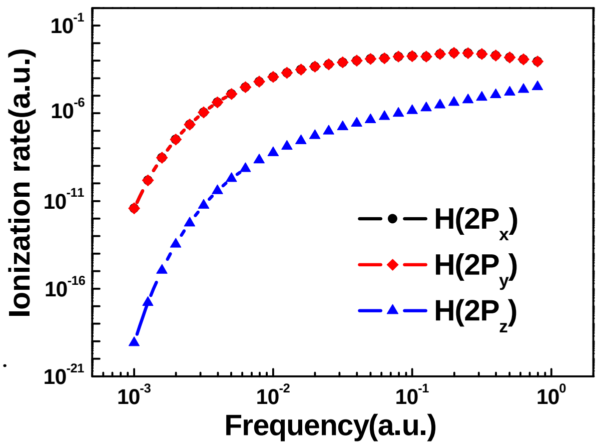

Figure 1: (Color online) Plot of three different hydrogen ionization rates for the linearly polarized laser field as a function of the laser frequency ω, with the Reiss intensity parameter z_1 fixed at 100.

In **Figure 2** the ionization rates are plotted as a function of the Reiss intensity parameter z_1 for ω=0.01 a.u.. These three ionization rates are shown with the same kind of lines as before. Each curve in **Figure 2** begins when ω=0.01 a.u. (we have numerically verified that, when the z_1 tends to high, the ionization rate of H($2p_x$) becomes an increasing function of z_1, the ionization rate of H($2p_z$) becomes a decreasing function of z_1).

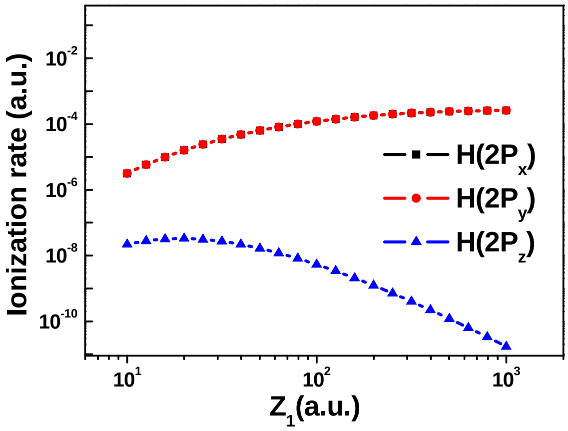

Figure 2: (Color online) Plot of three different hydrogen ionization rates for the linearly polarized laser field as a function of the Reiss intensity parameter z_1, with the laser frequency ω fixed at 0.01 a.u.

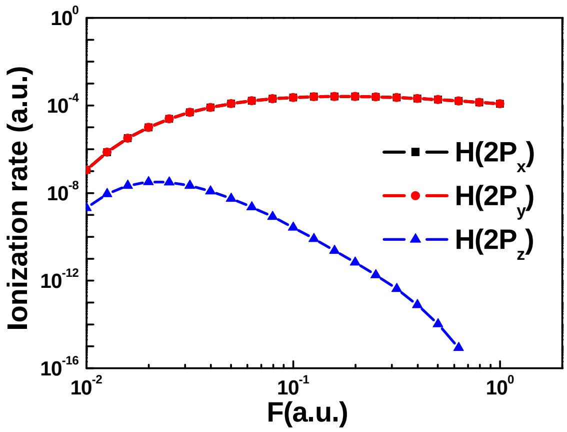

Figure 3: (Color online) Plot of three different hydrogen ionization rates for the linearly polarized laser field as a function of the amplitude F, with the laser requency ω fixed at 0.01 a.u. and the Reiss intensity parameter z_1 fixed at 100.

In **Figure 3** the ionization rates are plotted as a function of the amplitude F for z_1 =100, ω =0.01 a.u.. These three ionization rates are shown with the same kind of lines as before. Each curve in **Figure. 3** begins when z_1 =100, ω =0.01 a.u. (we have numerically verified that, when the amplitude F tends to high, the ionization rate of H($2p_x$) becomes an increasing function of amplitude F, the ionization rate of H($2p_z$) becomes a decreasing function of amplitude F). However, in the ionization rate of H($2p_x$) such a downfall is much weaker. In the VG ionization rate depends strongly both on frequency and intensity for the strongest laser fields. It always happens when the classical radius of motion of a free electron in the laser field is much larger than the radius of the atom: $\partial_0 = \frac{F}{\omega^2} \gg \frac{1}{Z} a.u.$ (Z=1 in figures 1-3).

4. Conclusion

In conclusion, we have derived the ionization rates of H($2p_x$), H($2p_y$) and H($2p_z$) using the VG KFR theory, which describe strong-field ionization in the linear polarized laser field. We have numerically compared ionization rates of H($2p_x$), H($2p_y$) and H($2p_z$). It is shown that there are no identical difference between the ionization rates of H($2p_x$) and H($2p_y$). In order to understand the origin of the discrepancy between the VG KFR theory and LG KFR theory for hydrogen atom, our derivation is based on the same approximations as used by JarosIaw H Bauer who have utilized the LG KFR theory and VG KFR theory for comparison. For H($2s$), H($2p_x$) and H($2p_y$) they have found a minimum in the LG distributions, whereas no minimum has been found in VG distributions. Ionization rates calculated in VG are orders of magnitude smaller than in the LG and show much greater stabilization. Some kind of envelope has been found in the LG for the curves showing ionization rate as a function of intensity of the laser field. One of the main results of this paper is Eq.(28), Eq.(36), Eq.(37), the VG KFR formula for the ionization rate for H($2p_x$), H($2p_y$) and H($2p_z$) of a hydrogen atom in the linear polarized laser field.

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