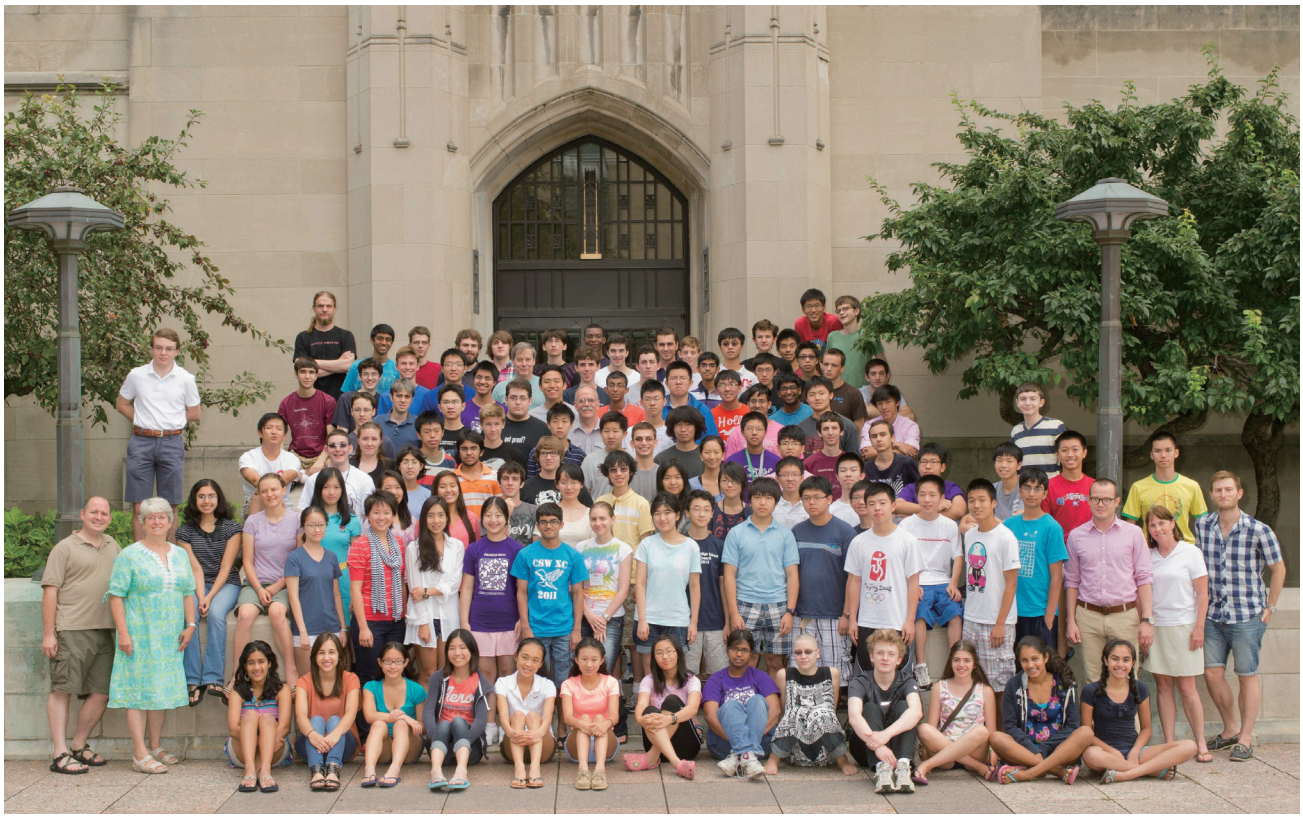

2012 年班级大合照

一段精彩的数学之旅——介绍一个高中数学夏令营

A Wonderful Math Program for High School Students

励容达

PROMYS 成立于 1989 年，其创始人是一群曾经参加并受益于 Ross Program（一个历史更加悠久的数学培养计划）的数学家，其目的是培养和发现有才华而又好学的数学学生。二十多年里经过 PROMYS 训练而如今已达研究生年龄的学生中，约有 50%已经获得或正在攻读博士学位，其中大多是在与数学有关的专业。他们中约有 100 位已成为大学教授，其中约有 70%是数学教授，其余的分布在计算机、物理、化学、生物、医学、法律、哲学及其它领域。

2011 及 2012 年夏天，我有机会去参加了这项活动，非常喜欢。在这里我把我的所见、所闻、所感写出来，与各位爱好数学特别是数论的朋友们分享。

“少年科学家数学项目”（简称 PROMYS）是一项每年在波士顿举行、时长 6 周面向全球高中生的夏令营活动。该活动始于 1989 年，由“罗斯项目”（阿诺德 • 罗斯在 1957 年创立）发展而来。

The Program in Mathematics for Young Scientists (PROMYS) is a six-week long summer camp in Boston, available to high school students from around the world. It started in 1989 based on the Ross Program (founded by Arnold Ross in 1957).

骨干教师（从左至右）；Margy Baruch, Glenn Stevens（主讲教师）, David Fried, and Steve Rosenberg

学习模式：

第一年参加的大约五十名新生将有机会学习数论的基础课程。这里的学习方式比较独特：它是以习题集为主导，每个课日（即非周末）都会布置一套习题并在下一个课日提交。一批来自美国名校（如哈佛、麻省理工、普林斯顿）数学专业的学生作为助教，平均每个助教负责四名学生。新生们将他们一天中的大部分时间都花在了习题集上，由助教批改。一个典型的问题集包括一段计算型的问题、一段严格证明型的问题、还有一段 PODASIPS（Prove or Disprove and Salvage if Possible，证明或反证及补救），另外还有一些自由探讨和查找资料的，围绕这 5 方面问题总共大约给出 17 个题目。

每天上午，知名的数论专家格伦 • 史蒂文斯（Glenn Stevens）教授都会进行九十分钟的授课，内容大致涵盖了三天前布置的习题集。这样做的目的是让学生在听老师讲解前自己动手试一试。

头三个星期，是训练一年级学生严格的数学证明与推理能力。先让他们把关于整数的公理压缩到简单几条，然后再证明有一定水平的结论（例如：存在 x , y 使得 $ax+by = 1$ 当且仅当 $gcd(a, b) = 1$，从 $a|bc$, $gcd(a, b) = 1$ 可以推出 $a|c$，整数的唯一分解，等等）。在第三个星期五有一次中考，包含 30 个问题，每题 12 分。通常学生能拿到 130 分左右。

接下来的三个星期，课程的内容会向不同方向展开：不同于整数的其它环的性质（整数添加$\sqrt{-5}$所形成的环，亦即 $\mathbb{Z}[\sqrt{-5}]$，等等）、连分数、关于凸体内格点个数的闵可夫斯基定理、默比乌斯变换、二次互逆定律以及其它种种。在最后一个星期会有一次大考。

每年还会有 20 个左右的“老学员”（第二或第三次来的学生），他们可以有较多的选择。每天一次的数论课他们

Content:

The 50 or so first-year students engage in learning about foundations of number theory. The format of this program is somewhat unique: Its dominant aspect is the problem sets, handed out each weekday and due the next weekday. Each student has a counselor, who is usually a student studying mathematics at some prestigious university (Harvard, MIT, Princeton, etc). Each counselor would be in charge of four students. The problem sets are marked by the counselors. The students spend most of their day working on the problem sets. A typical problem set would consist of a section on numericals, a section on rigor, a section of PODASIPS (Prove or Disprove and Salvage if Possible), exploration sections and possibly a reading search, totaling around 17 problems give or take 5 questions. There were 90-minute lectures every weekday by Professor Glenn Stevens, a well-known expert in number theory. The lectures are designed so as to approximately cover the content of the problem sets from three weekdays ago. This was to ensure that students would be able to have a go at the problems on their own first. During the first three weeks, first-year students develop their ability for mathematical rigor and proof by producing a reduced inventory of axioms of integers and progressing to prove statements up to certain level. (Examples: there exist x and y such that $ax+by = 1$ iff $gcd(a,b) = 1$), ($a|bc$, $gcd(a,b) = 1$ implies $a|c$, unique prime factorization…). On the Friday of the third week, there is a midterm test, which consists of about 30 problems worth 12 marks each. A typical score is 130. In the final three weeks, the content of the course branches out, examining properties of other rings (integers adjoint root -5, etc), continued fractions, Minkowski's theorem about convex, symmetric spaces centered in the origin of a lattice, Mobius inversion, quadratic reciprocity, and many other topics. There is a final exam in the last week.

The 20 or so returning students (those who have come back for a second or third year) have a number of options available to them. While they are required to attend the first-year lectures about number theory, they do not need to complete number theory problem sets. There are a number of courses available to returning students; these may or may not change each year. This year, the courses available, ranked from what was generally seen as 'easiest' to 'hardest', were Abstract Algebra (taught by Marjorie Baruch from Syracuse university), Geometry and Symmetry (Steven Rosenberg, Boston University), and the Analytic Class Number Formula(taught by Jared Weinstein, Boston University). The returning-student courses were different to the number theory courses in format, in that new concepts

2012 年老师、助教、工作人员大合照

必须参加，只是不需要做习题。他们可以参加若干专题课，这些课程每年或许会有些变化。今年（2012 年）有三门专题课，大家普遍认为从“易”到“难”的依次是：抽象代数——雪城（Syracuse）大学的马乔里 • 巴鲁克（Marjorie Baruch）教授主讲，几何与对称—— 波士顿大学的斯蒂芬 • 罗森伯格（Steven Rosenberg）教授主讲，理想类数的解析公式——波士顿大学的杰瑞德 • 温斯坦（Jared Weinstein）教授主讲。

开给“老学员”的课，在授课方式上与基础数论课不同。每天安排的作业是与当天授课内容同步的，而不像基础数论课有 3 天的延时。上述三门课，每周分别布置 3 次、2 次和 1 次作业。今年我上了抽象代数和理想类数的解析公式课。

在抽象代数课中，我们从群的定义开始，逐步进展到凯莱（Cayley）定理与西罗（Sylow）定理。在理想类数的解析公式课中，我们从黎曼 zeta 函数开始，渐进到理解理想数、狄利克雷特征，并最终找到了各种二次域和分圆域的理想类数。

were usually introduced synchronously in lectures and the homework, rather than on a three-day delay as in the case of number theory. Homework was assigned three, two and one times per week in the respective courses. I attended Abstract Algebra and the Analytic Class Number Formula courses this year. In the Abstract Algebra course, we started with the definition of a Group and progress to topics about the level of Cayley's Theorem and Sylow's Theorem. In the Analytic Class Number Formula course, we began with the meaning of the Riemann Zeta function, progressed to start understanding ideals, looked at Dirichlet characters, and found the class number of various quadratic fields and cyclotomic fields.

In the midterm and final exam, if any returning students did particularly well in previous years, they would be allowed to take more difficult exams called, respectively, the 'short', the 'super short', and the 'duper super short' due to the fact that they

在中考和大考的时候，如果哪个“老学员”在前一年考得特别好，那么就可以参加一些更难的考试，因为考题的数目与难度成反比，因此这些考试由简到难分别被称为：“短”、“特短”、“特特短”。

所有的学生都可以加入研究小组，每个研究小组由一个助教负责，通常有 4-5 个学生参加。

第一年的新生可以选择是否参加研究小组。如果参加，他们的研究课题会是比如分式线性变换、差分演算、分拆以及各种组合问题等。

“老学员”是必须参加研究小组的。 他们的研究课题会是，比如：根子系统与外尔群的表示（我们学习了什么是根式，什么是表示，然后设法去寻找生成 D4 和 F4 类的根系的所有表示的方法）。其它课题包括用随机对合来模拟素因子、对称多项式的形变以及抛物线上的“有理距离集”。

本 • 哈里斯（Ben Harris）博士是我所在研究小组的指导员（负责给研究小组成员出题，并且不时给予指导），他是麻省理工 2011 年毕业的博士。我们的助教是袁乔初，今年刚从麻省理工毕业。那次 PROMYS 临近结束前，我们小组做了一个时长一小时的研究报告，并将我们的结果用 LaTex 制成了论文。

助教们还开一些小课，向全体 PROMYS 学生开放。小课的时间是每次一小时，介绍各种各样有趣的课题。比如有一个课程是关于图上的游走。助教告诉我们有些问题（比如判定斐波那契数列或是计算由 A，C，T 组成而长度为 n 且不包含“CAT”的字符串有多少）可以如何表达为图上的游走的计算，而计算图上的游走又可以如何表达为求邻接矩阵的幂的问题，以及如何用特征值和特征向量来快速计算某些矩阵的幂。在另一堂课上，助教花了整堂课的时间来定义 p-adic 数。

PROMYS 也给助教们提供了学习课程，只是具体安排我不太清楚。 但无论如何，在波士顿的那 6 个星期里，助教们也在努力钻研，刻苦攻关。

特别优秀的学生可以在下一年回来做“小助教”。

PROMYS 还邀请专家举办客座演讲，比如有斯蒂芬 • 沃尔夫勒姆（Stephen Wolfram，Mathematica 和 Wolfram Alpha 的发明人），本 • 哈里斯（麻省理工博士，曾参加过 PROMYS 的学员），还有一些谷歌公司的专家等。

在 PROMYS 还有一些像是高中老师的人，他们似乎是来学习好的教学方法的，我和他们接触不多。

对我个人而言 PROMYS 的优点在于：

一般情况下我不会逼着自己去学习数学。可是在 PROMYS，看到这么多天智聪慧的同学和老师对数学这么感兴趣，又那么钻研，深深受到影响，在周围环境的带动下自然而然地沉下心来，认真学习而不觉辛苦。PROMYS 是

progressively involved fewer questions of greater difficulty.

There are also research labs available to all students. There is a counselor assigned to each research lab, and each research lab usually has 4 or 5 students.

Research labs were optional for first-year students. Examples of topics of research labs for first-year students include fractional linear functions, calculus of finite differences, partitions, and combinatorial problems.

Research labs were mandatory for returning students. Examples of topics of research labs for second-year students included Root Subsystems and Weyl Group Representations, in which we learned what root systems were and what representations were, and looked for ways to generate representations of root systems of type D4 and F4. Other topics are Modeling prime divisors by random involution, Deformations of symmetric polynomials, and Rational-distance sets on parabolas. In my research lab, our mentor (the person who posed the question, and periodically turned up to help us) was Dr. Ben Harris, who had graduated with a PhD from MIT in 2011. Our counselor was Qiaochu Yuan, who graduated from MIT recently as well. Near the end of the program, we did an hour-long presentation of our research lab, and produced a write-up of our results using LaTeX.

There were also mini-courses taught by counselors, available to all people at PROMYS. These were hour-long lectures about all sorts of interesting topics. One example of a course is walks on graphs: In the course, the counselor taught students how certain problems, such as determining Fibonacci numbers or finding the number of letter combinations of length n consisting of letters A,C,T did not contain "CAT", could be expressed as counting walks on graphs, and how, in turn, counting walks on graphs could be expressed in terms of exponentiation of adjacency matrices, and how quick calculations of exponentiations of certain matrices was possible by using eigenvalues and eigenvectors. In another course, the counselor spent the whole time to define p-adic numbers.

There were also counselor courses, the structure of which I am not too sure about. Whatever it was, during the 6 weeks in Boston the counselors were deeply engaged in mathematics at a level that would be challenging to them as well.

Exceedingly good students were offered the opportunity to return the next year as 'junior counselors'.

There were also guest lectures. Stephen Wolfram (Creator of Mathematica and Wolfram Alpha), Ben Harris (Past student and PhD from MIT), Some Google person, Amanda Beeson

作者（左）与他的同学在一起

不允许带计算机的，我们学习效率这么高，可能要归功于这个规定。这里有休息室、教室以及其它可供合作交流的空间场所，对于我们的学习很有帮助。

在开幕式上，格伦·史蒂文斯教授表达了如下的观点：“我们选择助教时，看的不是他们的教学水平，而是他们的数学能力。巧的是这些老师恰好也很会教书。”这当然是个见仁见智的问题，但我觉得作为助教，其数学能力与教学及交流能力同样重要。在 PROMYS 有不少同学是不太善于交际的，所以热情、主动的助教对那些内向的学生将会有所帮助。

录取：

PROMYS 的网页上有整套的申请资料。 他们选择学生时更看重其解决问题的能力，而不仅仅是准确的答案。PROMYS 采取滚动招生的方式录取学生。

经费来源：

PROMYS 的经费主要来源于几个大的机构及私人捐助者。高级讲座系列是由 Clay 数学所赞助的，他们相信 PROMYS 是在培育未来的数学家和科学家。第一年学生的学费是 2700 美元，“老学员”则是 2200 美元，每周 19 顿的餐费也包括在内。美国数学学会和私人捐款也提供了几项奖学金。PROMYS 在其网页上这样声明：“没有人会因为经济的原因而来不了这里。”

(Rochester), were some of the lecturers.

There was a group of people, seemingly high school teachers, who appeared to be there in order to learn good teaching methods. I didn’t interact with them much.

Things that made it good, in my opinion:

I am not usually capable of forcing myself to study loads of mathematics. A major part of PROMYS that empowered me to sit down and work was that there was a 'critical mass' of students and counselors who were mostly really good at, and interested in, mathematics. There was a positive peer pressure effect that just seemed to make it really easy to convince you to work hard. Computers were disallowed in PROMYS; this may have contributed to productivity. There were also lounges, classrooms and other collaborative study spaces available, which definitely helped with studying.

At the start of the program, professor Glenn Stevens said something along the lines of, "Our counselors are not chosen by their teaching ability. They are chosen by their mathematical ability, and it just so happens that they are excellent teachers."Make of that what you will, but I did think that it was important that the counselors were not only mathematically talented, but also genuinely good at teaching and communicating. There were many asocial students at PROMYS, so I think it helped that the counselors were the kind of people who would reach out to those students as well.

Admission:

The questions for the student application packet are available on the PROMYS website. PROMYS appeared to pick students by problem-solving ability rather than necessarily the actual solutions, and had a rolling admissions process.

Funding:

PROMYS is supported by a number of organizations as well as some private donations. The Advanced Seminars are sponsored by the Clay Mathematics Institute, in the belief that PROMYS nurtures future mathematicians and scientists. The tuition is USD2700 for first-year students and USD2200 for returning students. 19 meals per week are included in the fee. Several scholarships are available with support from the American Mathematical Society and private donors. The PROMYS website states that "no student should be unable to attend for financial reasons".