布拉格天文钟的数学原理

The Mathematics Behind Prague's Horologe

M. Krizek （捷克）
A. Solcova (捷克)
L. Somer （美国）

一、引言

欢迎来到捷克共和国的首都——布拉格。这个中欧城市不但拥有“千塔之城”的美誉，而且更被联合国教科文组织列入世界文化遗产。不少举世闻名的数学家、物理学家和天文学家都在这片土地上度过光辉的岁月，留下许许多多永不磨灭的历史印记。其中的代表人物包括文艺复兴时期的意大利哲学家乔尔达诺•布鲁诺(Giordano Bruno)、丹麦天文学家第谷•布拉赫 (Tycho Brahe)、其助手德国天文学家约翰尼斯•开普勒 (Johannes Kepler)、波希米亚数学家伯纳德•波尔查诺 (Bernard Bolzano)、法国数学家奥古斯汀•柯西 (August Cauchy)、挪威数学家尼尔斯•亨利克•阿贝尔 (Niels Henrik Abel)、奥地利数学及物理学家克里斯提昂•多普勒 (Christian Doppler)、奥地利物理学及哲学家恩斯特•马赫 (Ernst Mach)、相对论创立人德国理论物理学家阿尔伯特•爱因

1. Introduction

Welcome to Prague — the capital of the Czech Republic, called the “City of a Hundred Towers”, located in central Europe, and designated as a World Heritage site by UNESCO. Many famous mathematicians, physicists and astronomers have spent here very fruitful and creative years, and left unforgettable traces in Prague, in particular, Giordano Bruno, Tycho Brahe, Johannes Kepler, Bernard Bolzano, August Cauchy, Niels Henrik Abel, Christian Doppler, Ernst Mach, Albert Einstein and his mathematical colleague Georg Pick who was one of the people who taught him the tensor calculus. During their stays in Prague the above-mentioned scientists developed several fundamental mathematical and physical theories and engaged in related activities. For instance, in the beginning of the 17th century Kepler formulated the first two of his three laws of planetary motion based on Tycho Brahe's observations. In the first half of the 19th century Bolzano constructed a nondifferentiable continuous function (of a fractal character) and wrote a treatise on infinite sets entitled Paradoxes of infinity (1851). In 1842 Doppler, professor of mathematics at the Prague Technical University, first lectured about his

斯坦 (Albert Einstein) 和他的大学同僚乔治•皮克 (Georg Pick)，而皮克是教授爱因斯坦张量微积分的数学家之一。上述科学家在布拉格生活期间，建立了几个对后世影响深远的数学和物理学理论，并且从事相关之研究工作。十七世纪初，开普勒根据第谷•布拉赫的观测结果，提出行星运动三大定律之第一、第二定律。十九世纪上半叶，波尔查诺给出了一个既有分形特征，又不可微分的连续函数，并且以无限数集 (infinite sets) 为题撰写了《无穷的诡论》(Paradoxes of Infinity, 1851)。1842年，布拉格理工大学数学教授多普勒在布拉格Ovocný trh(图1)的查理大学，首次就其创立的效应 (后来称为“多普勒效应”) 发表公开演讲。1911年至1912年，爱因斯坦在布拉格德国大学 (Prague German University) 担任理论物理学教授，醉心于广义相对论的研究工作。甚至著名捷克作家卡雷尔• 恰佩克 (Karel Capek) 也是在布拉格发明“robot”这个字 (捷克语Robota，意谓劳役、苦工)。本文后面会简单介绍这些与布拉格息息相关的伟人纪念碑和雕塑。接下来，本文集中讨论布拉格旧城广场中心的一个著名建筑物，分析其中有趣的数学问题。

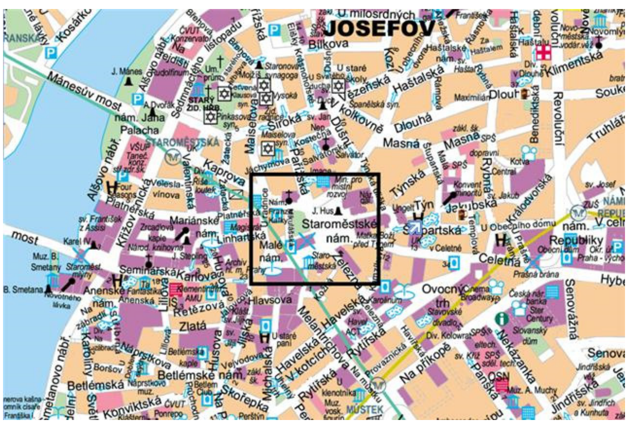

Fig.1 布拉格旧城区中心地图

Map of the Old Town in the center of Prague

布拉格旧城区中心(图1-2)有一个古色古香的天文钟 (捷克语orloj，英语horologe)。无论是一般游客，或是热爱数学的人，都会慕名而来一睹这个举世稀有的珍品。本文将揭示天文钟与三角形数之间鲜为人知的关系，探讨三角形数的特性，以及这些特性如何提升大钟的准确度。

布拉格天文钟的数学模型设计来自简•安卓亚 (Joannes Andreae，捷克语Jan Ondrejuv，生于1375年，

effect (later called the Doppler effect) at Charles University at Ovocný trh (see Figure 1). Einstein, while a professor of theoretical physics at the Prague German University, worked on his theory of general relativity in 1911–1912. The famous Czech writer Karel Čapek invented the term “robot” in Prague. Before briefly detailing plaques, statues, and other memorials to these personalities of Prague at the end of this paper, we discuss some unexpected mathematics associated with a prominent building at the heart of the Old Town Square of Prague.

In the center of Old Town in Prague (see Figures 1 and 2), there is an astronomical clock (called “orloj” in the Czech language and horologe in English) — an interesting rarity often visited by many tourists, not only mathematical tourists. We found that there is a surprising connection between this clock and triangular numbers. In this article we take notice of special properties of these numbers that make the regulation of the bellworks more precise.

Fig 2. 旧城广场天文钟的位置

Location of the astronomical clock at the Old Town Square (Staroměstské náměstí)

The mathematical model of the astronomical clock of Prague was invented by Jan Ondřejův, called Šindel (Joannes Andreae, cca 1375–1456), a professor at Prague University founded in 1348 by the Emperor Charles IV. In 1410, Šindel was the rector there. The astronomical clock was realized by the skilled clockmaker Mikuláš (Nicholas) from Kadaň in 1410, i.e., exactly 600 years ago.

The astronomical clock of Prague is placed inside an almost 60 m high tower of the Old Town City Hall. The clock has two large dial-plates on the south wall of the tower (see Figure 3). Over the centuries the construction of the clock has been renovated several times, for example, by the clockmaker Jan from Růže (called Master Hanuš) around 1490. A memorial plaque devoted to the creators of the clock is on the left of the lower dial-plate.

The upper dial-plate of the astronomical clock is an astrolabe controlled by a clockwork mechanism. It represents a stereographic

卒于1456年）。安卓亚又名辛蒂尔（Šindel），在国王查理四世于1348年所创办的布拉格大学任教。1410年，辛蒂尔当上大学院长，天文钟的设计意念终于通过卡丹市（Kadaň）的钟匠密库拉斯（捷克语Mikulás，即英语Nicholas）得以实现。

布拉格天文钟设于旧城市政厅一座约六十米高的钟楼内，而两个大钟盘（图3）则镶嵌在钟楼南面的外墙上。六百年来，天文钟经历过几次大型翻新，其中一次约在1490年，由克伦洛夫城（Ruze）的钟表工匠简恩（Jan，又名哈劳斯大师Master Hanus）带领进行。天文钟下钟盘的左方特别设置一面纪念碑，用来表彰这些钟匠们的付出和贡献。

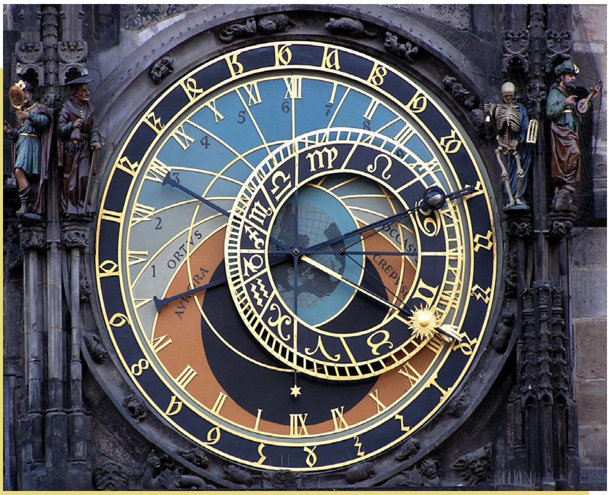

Fig. 4　天文钟的上钟盘
The upper dial-plate of the astronomical clock

Fig 3.　布拉格天文钟的两个钟盘
The dial-plates of the astronomical clock of Prague

projection of the celestial sphere from its North Pole onto the tangent plane passing through the South Pole. The center of the dial-plate thus corresponds to the South Pole of the celestial sphere. The smallest interior circle around the South Pole illustrates the Tropic of Capricorn, whereas the exterior circle illustrates the Tropic of Cancer. The concentric circle between them corresponds to the equator of the celestial sphere (see Fig 4).

An important property (known already to Ptolemy) of the stereographic projection is:

Any circle on the sphere which does not pass through the North Pole is mapped onto a circle as well.

Therefore, the ecliptic on the celestial sphere is projected on a circle, which is represented by the gilded ring with zodiac signs along the ecliptic. However, its center is not the South Pole, but the ring eccentrically rotates around this pole (see Figure 4). The astronomical clock also shows the approximate position of the Sun on the ecliptic, the motion of the Moon and its phases, and the rising, culmination and setting of the Sun, the Moon and zodiac signs.

The gilded solar hand indicates the Central-European time (CET) in the ring of Roman numerals. Note that the difference between CET and the original Prague local time is only 138 seconds. The clock-hand with a small gilded asterisk shows the sidereal time (see Figure 4). Twenty four golden Arabic

天文钟的上钟盘是一个靠发条机制控制而运作的星盘，象征天球 (celestial sphere) 由其北极通过南极落在切平面上的球极平面投影 (stereographic projection)。钟盘的中心点相当于天球的南极，南极四周的最小内圆代表南回归线，而外圆则代表北回归线，两者之间的同心圆相当于天球赤道 (图4)。

球极平面投影有一基本性质 (由古希腊天文学家托勒密Ptolemy提出):

球体上所有异于北极的圆，经过球极平面投影法，在平面上的投影也是一个圆。

因此，天球黄道的投影也是圆形，用刻有十二星座图案的镀金钟圈表示。虽然天球黄道的中心点并不是南极点，但是镀金钟圈却神奇地绕着南极点转动(如图4所示)。此外，天文钟也指出了太阳在黄道上的大概位置、月球的运动和月相，以及日、月和十二星座各自的出、落和中天时间。

镀金太阳指针在罗马数字钟圈上转动，显示的是中欧时间 (Central European Time，简称CET)，值得留意的是，中欧时间和原来的布拉格当地时间相差只有138秒。旁边那枝镀金星星指针所显示的是恒星时(sidereal time)。最外那个钟圈上有金制的阿拉伯数字1至24，标示从日落起计算的古捷克时间；而下方黑色的阿拉伯数字1至12，是用来标示早在巴比伦时代已经开始使用的行星时间 (planetary hours)。行星时间则由日出开始算起，与古捷克时间的计算方法相反。

钟盘下方的黑色圆形部分代表天文曙暮光 (astronomical night)，即太阳处于地平线下18度的时段；外围的棕色部分象征黎明和黄昏 (AVRORA 和 CREPVSCVLV 标志着白昼和夜晚)，而 ORTVS 和 OCCASVS 则代表日出和日落。

天文钟的主装置有三个同心大齿轮，每个直径为116厘米，最初由三个各有24齿的小齿轮驱动。第一个大齿轮有365齿，每个恒星日 (即23小时56分4秒) 推动星座钟圈转一周。第二个大齿轮有366齿，每一平太阳日 (mean sun day) 推动太阳指针转动一圈。由于地球环绕太阳的的公转轨道是椭圆形而非圆形，因此太阳在天球上的运动速度不均一。现时，星座钟圈的位置每年要经人手调校两次。第三个大齿轮有379齿，推动月

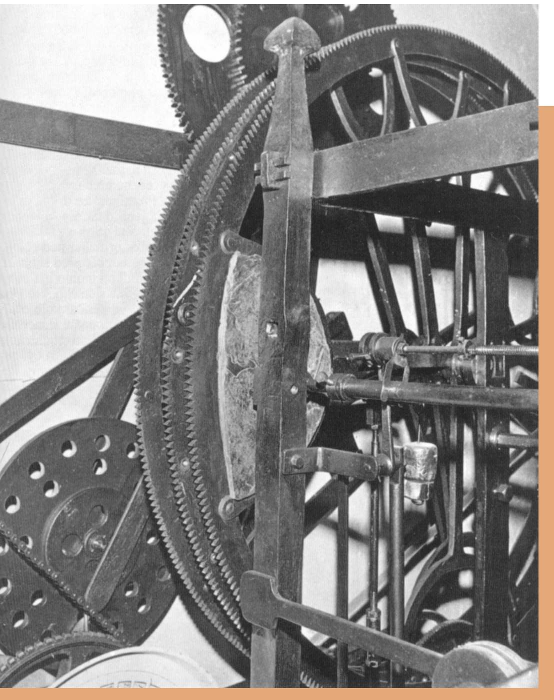

Fig. 5 主钟的详图(后面那三个同心大齿轮十五世纪初开始运作)
A detail of the main clock. The three large concentric gears behind are from the beginning of the 15th century

numerals are used for the ancient Czech time measured from sunset. Twelve black Arabic numerals denote planetary hours of the Babylonian time measured from sunrise.

The black circular area at the bottom of the dial-plate corresponds to the astronomical night, when the Sun is lower than 18° below horizon. The brown area stands for twilight (AVRORA in the morning and CREPVSCVLVM in the evening). Sunrise is denoted by ORTVS and sunset by OCCASVS.

In the main clockwork, there are three large original concentric gears of the same diameter 116 cm (see Figure 5) which were originally driven on one axis by three pinions, each with 24 teeth. The first gear has 365 teeth and turns round the zodiac ring once per sidereal day (23 hr 56 min 4 s). The second gear, which has 366 teeth, drives the solar pointer and turns round once per mean solar day. Since the true orbit of the Earth is elliptic, the Sun does not move uniformly on the celestial sphere. Therefore, the position of the zodiac ring is at present slightly corrected manually twice per year. The third gear, which has 379 teeth, drives the Moon's hand and rotates according to the mean apparent motion of the Moon. The lunar pointer is also at present manually corrected due to the elliptic orbit of the Moon. The lunar pointer (see Figure 4) is a hollow sphere with a hidden mechanism inside that displays the phases of the Moon. It was developed in the 17th century

亮指针根据月球的视运动（mean apparent motion）而转动。因为月球轨道同样是椭圆形，所以月亮指针也需要不时以人手校准。月亮指针（图 4）其实是个空心球体，内藏机关，可展示月相。这个指针设计于十七世纪，转动的动力来自椭圆环圈的运动。

下面的钟盘是个月历钟，上面有十二幅由马内斯（Josef Mánes）绘画的饼图画，每一年转一周，最上的钟针标示一年中的某一日，同时亦提供取名日（namedays）等信息。

二、布拉格天文钟隐藏着怎样的数学原理？

下述例子诠释了十五世纪钟表工匠的精湛技术。天文钟的机械组件里，有一个大齿轮，它的圆周上有24道齿槽，齿槽间的距离随圆周逐渐递增（图6-7）。这个装置使大钟每天重复地按时敲打一至二十四下。与大齿轮连着一个辅助齿轮有六道齿槽，齿轮圆周按照“1，2，3，4，3，2”的比例分成六段。这几个数字合起来构成了一个循环周期，令齿轮不断地重复转动。这六个数字的和是15。

每到整点，扣子便会升起，大小齿轮便会运转。耶稣十二门徒的小木偶会通过钟面两侧的小窗口列队绕行

and draws energy from the movement of the Moon hand.

The lower dial-plate with 12 round pictures by Josef Mánes (see Fig 3) is a calendar. It turns round only once per year. The clock-hand on the top shows the particular day of the year. It also provides information about namedays and other items.

2. What mathematics is hidden behind the astronomical clock of Prague?

The ingenuity and skill of clockmakers of the 15th century can be demonstrated by the following example. The bellworks of the astronomical clock contains a large gear with 24 slots (the first two are connected) at increasing distances along its circumference (see Figs 6 and 7). This arrangement allows for a periodic repetition of 1–24 strokes of the bell each day. There is also a small auxiliary gear whose circumference is divided by 6 slots into segments of arc lengths 1, 2, 3, 4, 3, 2 (see Figs 6 and 7). These numbers constitute a period which repeats after each revolution and their sum is s = 15.

At the beginning of every hour a catch rises, both gears start to revolve, the 12 apostles appear and transit through two windows in sequence, and finally the bell chimes. The gears stop when the catch simultaneously falls back into the slots on both gears. The bell strikes

$$1 + 2 + \cdots + 24 = 300$$

times every day. Since this number is divisible by s = 15, the small gear is always at the same position at the beginning of each day.

The large gear has 120 interior teeth which drop into a pin gear

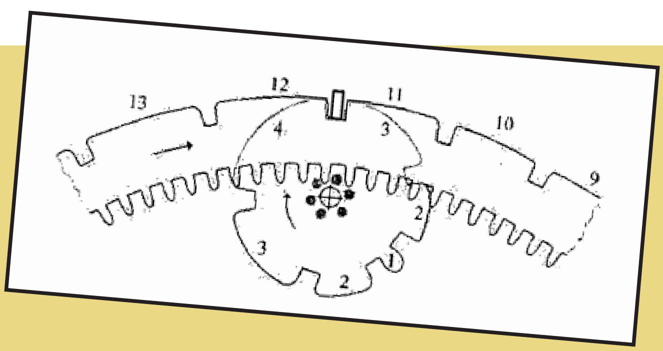

Fig.6　大齿轮外显示的数字表示大钟在整点时敲打的次数——“9，10，11，12，13……”。后面小齿轮的弧长比例为 1，2，3，4，3，2。上面的小长方形代表卡在两个齿轮之间的扣子。小齿轮转动时，通过其齿槽产生一个循环序列，其总和相等于整点时大钟敲打的次数——1，2，3，4，5=3+2，6=1+2+3，7=4+3，8=2+1+2+3，9=4+3+2，10=1+2+3+4，11=3+2+1+2+3，12=4+3+2+1+2，13=3+4+3+2+1，14=2+3+4+3+2，15=1+2+3+4+3+2……。

The number of bell strokes is denoted by the numbers ⋯ , 9, 10, 11, 12, 13, ⋯ along the large gear. The small gear placed behind it is divided by slots into segments of arc lengths 1, 2, 3, 4, 3, 2. The catch is indicated by a small rectangle on the top. When the small gear revolves it generates by means of its slots a periodic sequence whose particular sums correspond to the number of strokes of the bell at each hour:1, 2, 3, 4, 5 = 3 + 2, 6 = 1 + 2 + 3, 7 = 4 + 3, 8 = 2 + 1 + 2 + 3, 9 = 4 + 3 + 2, 10 = 1 + 2 + 3 + 4, 11 = 3 + 2 + 1 + 2 + 3, 12 = 4 + 3 + 2 + 1 + 2, 13 = 3 + 4 + 3 + 2 + 1, 14 = 2 + 3 + 4 + 3 + 2, 15 = 1 + 2 + 3 + 4 + 3 + 2 ⋯

一圈，然后钟声徐徐响起。待扣子回落在齿槽时，两个齿轮就会停止转动。大钟每天敲打的次数是1 + 2 + ⋯ + 24 = 300下。由于300能被15整除，所以小齿轮每天同一时间的位置都是不变的。

Fig. 7　天文钟详图中小齿轮的位置。图中的扣子卡在大齿轮的十八时与十九时之间的齿距上。

The location of the small gear. The catch is in the slot between the segments corresponding to 18 and 19 hours on the large gear. (Photo: Lukas Kalista)

大齿轮有120个内齿，啮合在一个针齿轮之中，针齿轮有六支围住小齿轮轴心的水平小横杆。大齿轮一天转一圈，而小齿轮则以高四倍左右的圆周速度一天转二十圈。这么一来，就算大齿轮出现磨损的情况，小齿轮都可以保持天文钟按刻报时的准确度。与此同时，小齿轮能够有效地使大钟在每天凌晨一时，只敲打一次。从(图7)所见，大齿轮的第一、二个齿槽之间并没有轮齿，即便有，也会因为太小而容易断开，所以，扣子只能够接触到小齿轮弧长为一的轮齿。

从文献 [2] 得知，上述的数列能够不断被建构出来，直至无限大。可是，并不是所有周期数列都拥有如此巧妙的总和特性。例如，我们可以很快便知道 1，2，3，4，5，

with 6 little horizontal bars that surround the center of the small gear (see Figures 6 and 7). The large gear revolves one time per day and therefore, the small gear revolves 20 times per day with approximately 4 times greater circumferential speed. Thus, the small gear makes the regulation of strokes sufficiently precise despite the wearing out of the slots on the large gear. Moreover, one stroke of the bell at one a.m. is due only to the movement of the small gear. In Figure 7 we observe that there is no tooth between the first and second slot of the large gear, since such a tooth would be extremely thin and thus, it could break. Therefore, in this case the catch is in contact only with the tooth of arc length 1 of the small gear, which makes the use of the small gear essential.

In [2] we show that we could continue in this way until infinity. However, not all periodic sequences have such a nice summation property. For instance, we immediately find that the period 1, 2, 3, 4, 5, 4, 3, 2 could not be used for such a purpose, since $6 < 4 + 3$. Also the period 1, 2, 3, 2 could not be used, since $2 + 1 < 4 < 2 + 1 + 2$.

The astronomical clock of Prague is probably the oldest [1, p. 76] still functioning clock that contains such an apparatus illustrated in Figure 6. Due to the beautiful summation property discussed above, Sloane in [3] and [4, A028355, A028356] calls the sequence 1, 2, 3, 4, 3, 2, 1, 2, 3, 4, … the clock sequence.

3. Connections with triangular numbers and periodic sequences

In this section we briefly mention how the triangular numbers

$$T_k = 1 + 2 + \cdots + k, \quad k = 0,\ 1,\ 2,\ \cdots$$

are related to the astronomical clock. We shall look for all periodic sequences that have a similar property as the clock sequence 1, 2, 3, 4, 3, 2, i.e., that could be used in the construction of the small gear. Put $N = \{1, 2, \cdots\}$.

The periodic sequence $\{a_i\}$ is said to be a Šindel sequence if for any positive integer k there exists a positive integer n such that

$$T_k = a_1 + \cdots + a_n, \qquad (1)$$

where the triangular number T_k on the left-hand side is equal to the sum $1 + \cdots + k$ of hours on the large gear, whereas the sum on the right-hand side expresses the corresponding rotation of the small gear (see Figure 8). In [2] we show that the above condition can be replaced by a much weaker condition based on only a finite number of k's: namely, the sequence $a_1, a_2, \cdots, a_p, a_1, a_2, \cdots$ with period length p is a Šindel sequence if there exists a positive integer n such that equation (1) holds for $k = 1, 2, \cdots, a_1 + a_2 + \cdots + a_p - 1$. This enables us to perform only a finite number of arithmetic operations to check whether a given period $a_1, \cdots, a_p$ yields a Šindel sequence. In [2] we also give an explicit algorithm for finding Šindel sequences.

4，3，2 不可用，因为 6 < 4 + 3；而 1，2，3，2 也不可用，因为 2 + 1 < 4 < 2 + 1 + 2。

布拉格天文钟很可能是世上现存少数装有（图 6）零件的大钟当中最古老的一个（文献 [1]，76 页）。正因上述完美的总和特性，美国数学家斯洛恩（Sloane，参考书目 [3] 及 [4，A028355，A028356] 的作者）把 1，2，3，4，3，2，1，2，3，4……称为时钟数列（clock sequence）。

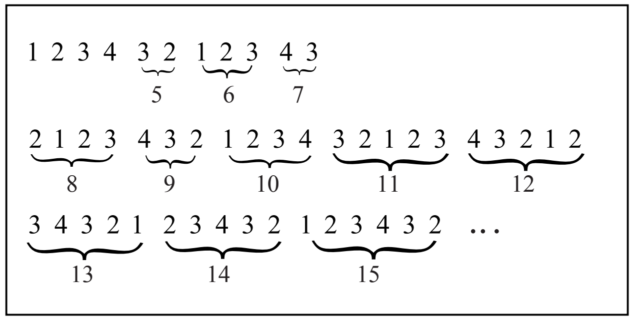

Fig. 8：每行上面的数代表了小齿轮的小节的长度；而下面的数表示第 k 个小时大钟敲打的次数。

The upper numbers denote lengths of segments on the small gear, whereas the lower numbers indicate the number of strokes at the k-th hour.

三、三角形数与周期数列的关系

本节简洁地论述三角形数

$$T_k = 1+2+\cdots+k, \quad k = 0,\ 1,\ 2,\ \cdots$$

与天文钟的关系，并找出所有跟时钟数列1，2，3，4，3，2拥有相同特性的周期数列，亦即可应用在小齿轮构造的周期数列。设 $N=\{1, 2, \cdots\}$。

若对任意正整数 k，存在一个正整数 n 使得

$$T_k = a_1 + \cdots + a_n \qquad (1)$$

成立，那么该周期数列 $\{a_i\}$ 会被称为辛蒂尔（Šindel）数列，其中等号左边的三角形数 T_k 等于大齿轮所有时刻的总和 $1+\cdots+k$，而右边数字的总和则表示小齿轮相应的转动圈数（图 8）。我们在参考资料 [2] 证明了，上述条件可被一个弱得多的条件所取代，只需要有限个 k，那就是，序列 $a_1, a_2, \cdots, a_p, a_1, a_2, \cdots$ 的周期长为 p，若存在正整数 n，使等式（1）对 $k = 1, 2, \cdots, a_1 + a_2 + \cdots + a_p - 1$ 成立，那么该数列就是辛蒂尔数列。这样便可在有限的运算次数中，检查某一周期 $a_1, \cdots, a_p$ 能否得出辛蒂尔数列。文献 [2] 也提供了查找辛蒂尔数列的显式算法。

四、其它具有数学及科学意义的名胜

离天文钟几米远的地方，竖立着一个爱因斯坦纪念牌，纪念他在1911年至1912年间，在旧城广场十七号暂住的岁月(见图9)。纪念牌旁边的哥德式教堂泰恩(Týn)，安放了建于1601年的第谷·布拉赫的墓冢，供游人参观。沿旧城广场向前走，一直到契里特纳大街

4. Further mathematical and scientific sights

Within a few meters from the horologe there is a memorial plaque devoted to Albert Einstein (see Fig 9) who often visited the house at Old Town Square no. 17 during 1911–1912. In the nearby gothic church called Týn we can find the tomb of Tycho Brahe from 1601. In Celetná street no. 25, which leads from the Old Town Square, there is a memorial plaque to Bernard Bolzano (see Fig 10). We also recommend visiting other plaques and busts of Albert Einstein (Viničná no. 7 and Lesnická no. 7), of Christian Doppler (Charles Square no. 20 and U Obecního dvora no. 7 – see Fig 11), of Johannes Kepler (Karlova street no. 4 and Ovocný trh no. 12/573), a large statue of Kepler and Brahe in Parléřova street no. 2 (see Fig 12), etc.

Prague is very popular among tourists and is celebrated as

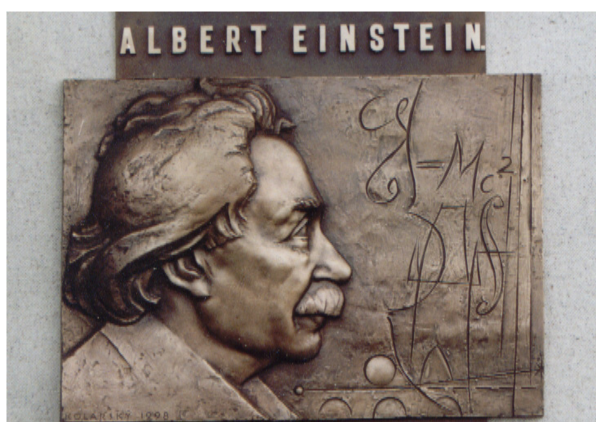

Fig .9 爱因斯坦纪念牌

Memorial plaque of Albert Einstein

(Celetná Street)25号，就会看见波尔查诺纪念碑(图10)。旧城广场附近还有其它科学家的纪念碑及半身塑像，同样也值得参观。这些人物包括爱因斯坦（Vinicná 7号、Lesnická 7号)、多普勒(查理广场20号、Obecniho dvora 7号；图11)、开普勒(查理大街 4号、Ovocny trh 12/573号)，以及位于Parlerova街2号的开普勒和布拉赫的大型雕像(图12)等。

作为旅游热点，布拉格拥有不同历史风格的建筑物，罗马式、哥德式、文艺复兴、巴洛克等建筑在市内比比皆是，因此被誉为最美、最浪漫的中欧城市之一。宏伟的布拉格城堡、艺术家和巴洛克雕塑处处可见的查理大桥(Charles Bridge)，还有泥巴妖怪勾勒姆(Golem) 传说的发源地——犹太区 (Jewish Town)。相传勾勒姆是由德高望重的犹太教师罗乌 (Rabbi Loew) 在十六世纪末左右制造，用来帮助当时居住在布拉格的犹太人对抗迫害。此外，布拉格也是欧洲的文化重镇，拥有浓厚的音乐传统，历史上曾有多部著名作品在此公演。1787年10月29日，享负盛名的莫扎特歌剧《唐璜》在查理大学附近的艾斯特歌剧院 (Estate Theatre) 首度公演。总括来说，不管你对数学是否感兴趣，布拉格都会给你带来无穷乐趣。

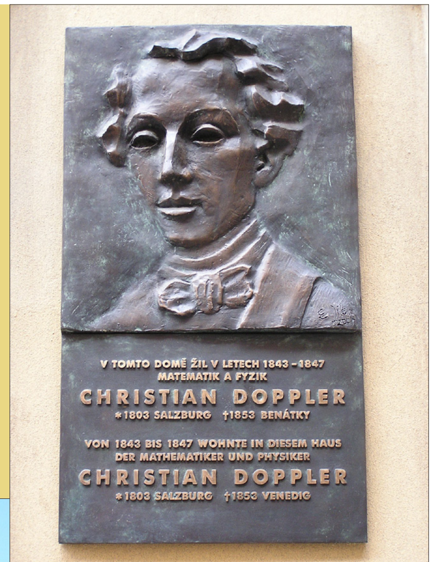

Fig. 11 多普勒纪念碑
Christian Doppler

Fig. 12 开普勒与布拉赫的大型雕塑
Johannes Kepler and Tycho Brahe

Fig. 10 波尔查诺纪念碑
Bernard Bolzano

one of the most beautiful and romantic cities in central Europe due to its varied architectural styles from many periods (Romanesque, Gothic, Renaissance, Baroque, …), the impressive Prague Castle, Charles Bridge with its many artists and Baroque statues, and the Jewish Town with its legend of the Golem, which was said to be created in the late 16th Century or early 17th Century by Rabbi Loew, the Maharal of Prague, to protect the Jews. Prague also has a rich musical tradition. For example, Mozart's opera Don Giovanni had its world premiere on October 29, 1787 at the Estates Theatre, which is adjacent to Charles University, where Jan Šindel taught about mathematics and astronomy. In addition, as demonstrated above, Prague also has much to offer to mathematical tourists.

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作者简介

Michal Krizek，捷克科学院数学研究所高级研究员，布拉格查理大学数学及物理学院教授。曾经多次访问中国，包括北京、天津、香港、湘潭等地。研究兴趣广泛，包括纯数学、计算数学和数学史。

Michal Křížek is a Senior Researcher at the Mathematical Institute of the Academy of Sciences of the Czech Republic and Professor at the Faculty of Mathematics and Physics of Charles University in Prague.

Alena Solcova，捷克理工大学信息技术学院计算机科学系助理教授，十七年前开始定期举办数学历史研讨会。

Alena Šolcová is an Associate Professor at the Department of Computer Science of the Faculty of Information Technology of the Czech Technical University in Prague. For more than 17 years she has held a regular seminar on the history of mathematics.

Lawrence Somer，华盛顿美国天主教大学数学教授，曾发表以数论为题材的论文。

Lawrence Somer is a Professor of mathematics at the Catholic University of America in Washington, D.C. He has published a number of papers on number theory.