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Coulomb divergence in S-matrix expansion of above-threshold ionization

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Abstract. Photoelectron spectrum of above-threshold ionization (ATI) is calculated using the first two orders of Coulomb-Volkov S-matrix series (Faisal, Phys. Rev. A 94, 031401 (2016)). Calculation shows that Coulomb divergence, due to singularity of forward scattering, still exists in the first two order terms for an unscreened Coulomb potential. This divergence can be removed by adding a decay factor into the phase of the S-matrix element which comes from depletion of the atomic ground state. We show that by using an example of hydrogen atom, the Coulomb-Volkov S-matrix series can successfully describe the energy spectrum of ATI and high-order ATI with the introduction of a deplete rate of the ground state.

Key words. above-threshold ionization, S-matrix thoery, Coulomb optential, rescattering.

1. Introduction

In the context of strong field-matter interaction, an increasing number of phenomena has been found and to be understood within the single-active-electron approximation. Such a phenomenon is above-threshold ionization (ATI) in which the ionized electron absorbs more photons than the minimum number necessary for ionization. With the discovery of ATI by Agostini et al. [1], intense-laser atom physics entered the non-perturbative regime where the force created by the strong laser field is comparable to or even larger than the Coulomb interaction between electrons and residual ion. The so-called strong field approximation (SFA) i.e., Keldysh-Faisal-Reiss (KFR) [2–4] theory has provided fruitful insights into a wide range of highly nonperturbative processes in intense fields.

The envelopes of energy spectra of ionized electron in ATI and high-order ATI(HATI) by a linearly polarized laser field typically consist of two regimes: one is a drop line with a cutoff near 2Up and a plateau extending to near 10 Up [5], with Up the electron's ponderomotive energy in the laser field. The low energy structure is attributed to those direct electrons that leave the laser focus without further interaction with their parent ion and decreases exponentially with increasing energy above 2Up. The high energy plateau, with lower yield by several orders of magnitude, is attributed to electrons that may backscatter off the parent ion and be accelerated again by the laser field reaching a kinetic energy of up to 10 Up.

Over the past several decades, however, plentiful theoretical researches mostly based on the KFR model with a plane-wave Volkov state obviously do not take into account the effect of the long-range Coulomb interaction in the ionization final state [6–10]. Therefore, originally the plane-wave KFR model has been explicitly proposed for the problem of electron detachment of negative ions [11] for which in the final state there is no Coulomb interaction between the detached electron and the residual neutral atom. The previous heuristic attempting to consider Coulomb corrections (e.g. [12–14]) had provided only a common overall enhancement factor that enlarges the total rate. Recently, a complete strong field S-matrix expansion that accounts for the final state Coulomb interaction in all orders is proposed explicitly to solve this long-standing problem [15].

The divergence of the Coulomb scattering amplitude in above-threshold ionization (ATI) is another open problem. For the long-range Coulomb potential, the forward-scattering cross section is large (actually, divergent), and this divergence is especially strong for Eret [16], where Eret denotes the return energy of electron before occurs rescattering with parent ion. This allows the low-energy-structure (LES) [17–19] to rise above the contribution of the forward rescattering electrons [20], i.e. p = k, where p, k are the final momentum and intermediate momentum in between ionization and recollision of the electron, respectively. Recently, in Ref. [21], an improved SFA-based calculation [22] in which the divergence caused by the Coulomb rescattering is removed by considering the depletion rate of the ground state in the energy denominator was presented.

A good agreement between theory and experiment of LES, demonstrated in Ref. [21], showed that the long-range Coulomb potential plays a major role on the LES, and Coulomb divergence mainly results from forward rescattering which is normally encountered in the high-order terms of the S-matrix expansion. In other words, Coulomb divergence is inevitable if we desire to obtain the structure of high energy plateau.

Here, in this paper, we combine the Coulomb-Volkov S-matrix series and the depletion rate of the ground state to describe the energy spectra of ATI and HATI without introduction of a suitable screening constant. This paper is organized as follows: In Sec.2 we introduce the complete Coulomb-Volkov S-matrix series for the ATI and HATI process, while in Sec.3 we present calculation of the transition amplitude for the ATI and HATI process in the case of a linearly polarized, infinitely long laser pulse. The method for calculation of this amplitude also can refer to [8, 23]. In Sec.4, we show our results and discussions. Finally, conclusions and comments about the physical meaning and importance of the results obtained are presented in Sec.5. The atomic system of units ($\hbar = |e| = m = 4\pi\varepsilon_0 = 1$) is used.

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2. Coulomb-Volkov S-matrix series

The time-dependent Schrödinger equation of the laser-atom interaction for hydrogen atom in the velocity gauge is

$$\left(i\frac{\partial}{\partial t}-\frac{\hat{p}^2}{2}+\frac{1}{r}-V_i(t)-V_{sr}(r)\right)|\Psi(r,t)\rangle=0, \tag{1}$$

where $V_i(t)=\left(-\vec{A}(t)\cdot\vec{p}+\vec{A}^2(t)/2\right)$, $\vec{A}(t)=A_0\cos\omega t\vec{e_z}$ is the vector potential of the laser field, and $\vec{p}=-i\nabla$, Vsr(r) is the short-range potential. For the final state, we take account of the long-range Coulomb potential interaction. The Coulomb-Volkov final state contains stationary Coulomb wave [24]is written by

$$\Phi_{\vec{p}}(r,t)=\phi_{\vec{p}}(r)e^{-i\int^t\left[\left(\vec{p}-\vec{A}(\tau)\right)^2/2\right]d\tau}, \tag{2}$$

where stationary Coulomb wave [25]

$$\phi_p(r)=\frac{1}{(2\pi)^{3/2}}e^{\pi/2p}\Gamma(1+i/p)e^{i\vec{p}\cdot r}$$
$$\times F_1\left(-i/p,1,-i(pr+\vec{p}\cdot\vec{r})\right), \tag{3}$$

Thus, we get the S-matrix element [9]:

$$S_{fi}=\langle\Phi_{\vec{p}}(t)|\Psi(t)\rangle$$
$$=\langle\Phi_{\vec{p}}(t)|\phi_i(t_1)\rangle-i\int dt_1\langle\Phi_{\vec{p}}(t_1)|V_i(t_1)|\phi_i(t_1)\rangle$$
$$-i\int\int dt_2dt_1\langle\Phi_{\vec{p}}(t)|V_{CV}(t_2)|G(t_2,t_1)V_i(t_1)|\phi_i(t_1)\rangle, \tag{4}$$

where,$|\phi_i(t)\rangle$ is the initial state wave function of hydrogen atom, t1,t2 denote the ionization time and the rescattering time, respectively. While $V_{CV}(t)=\left[-\vec{A}\cdot(\hat{p}-\hat{h})+V_{sr}(\boldsymbol{r})\right]$ is the corresponding rest interaction in the final state, $\hat{h}=\sum_s|\phi_s\rangle\vec{s}\langle\phi_s|$, where $\sum_s(\cdots)\equiv\int d^3s(\cdots)$, and $|\phi_s\rangle$ stands for the Coulomb continuum waves with momentum s. G(t1,t2) is the full propagator in terms of the Volkov propagator G_{Vol} and the intermediate interaction $V_0(\tau)$, as follows:

$$G(t_2,t_1)=G_{Vol}(t_2,t_1)+\int d\tau\,G_{Vol}(t_2,\tau)V_0(\tau)G(\tau,t_1)$$
$$=G_{Vol}(t_2,t_1)+\int d\tau\,G_{Vol}(t_2,\tau)V_0(\tau)G_{Vol}(\tau,t_1)$$
$$+\int d\tau' d\tau\,G_{Vol}(t_2,\tau')V_0(\tau')G_{Vol}(\tau',\tau)V_0(\tau)$$
$$\times G_{Vol}(\tau,t_1)+\cdots \tag{5}$$

where (for details see ref. [15]):

$$G_{Vol}(t_2,t_1)=-ie^{-i\int_{t_1}^{t_2}[(\vec{p}-\vec{A}(\tau)^2)/2]d\tau}\int d\vec{p}\,|\,p\rangle\langle p\,| \tag{6}$$

where, $|\vec{p}\rangle=\frac{1}{(2\pi)^{3/2}}e^{i\vec{p}\cdot\vec{r}}$ is plane wave function, and the rest interaction is:

$$V_0(t)=H(t)-H_{Vol}(t)=[-\frac{1}{r}+V_{sr}(\mathbf{r})]. \tag{7}$$

Finally, collecting the resulting terms from Eq.(4) explicitly, we obtain the all-order Coulomb-Volkov S-matrix series of ATI:

$$S_{fi}=S_{fi}^{(1)}+S_{fi}^{(2)}+\cdots$$
$$=-i\int dt_1\langle\Phi_{\vec{p}}(\vec{r}_1,t_1)|-\vec{A}(t_1)\cdot\vec{p}+\vec{A}^2(t_1/2)|\phi_i(\vec{r}_1,t_1)\rangle$$
$$-i\iint dt_2dt_1\langle\Phi_{\vec{p}}(\vec{r}_2,t_2)|-\vec{A}(t_2)\cdot(\hat{p}-\hat{h})+V_{sr}(\vec{r})\,|$$
$$G_{Vol}(\vec{r}_2,t_2,\vec{r}_1,t_1)\times\left(-\vec{A}(t_1)\cdot\vec{p}+\vec{A}^2(t_1)/2\right)|\phi_i(\vec{r}_1,t_1)\rangle$$
$$+\cdots \tag{8}$$

It should be noted that the S-matrix series introduced above, like most other well-known S-matrix series, have not been proven to be convergent [15]. Then we demonstrate that $|S_{fi}^{(2)}|^2$ is larger than $|S_{fi}^{(1)}|^2$ by calculating the first two orders of Coulomb-Volkov S-matrix series of ATI for hydrogen atom in the infrared laser field. It can be seen easily from Eq.(8) that the first two terms of the Coulomb-Volkov S-matrix series take into account both the "direct" electron that departs from the atom without any further interaction with the binding potential as well as those electrons rescattering with the ionic core.

3. Calculation of the transition amplitude

The first term of the Coulomb-Volkov S-matrix series does not contain the forward scattering, so there is no Coulomb divergence.

$$S_{fi}^{(1)}=-i\int dt_1\langle\Phi_{\vec{p}}(\vec{r}_1,t_1)|\left(-\vec{A}(t_1)\cdot\vec{p}+\vec{A}^2(t_1/2)\right)|\phi_i(\vec{r}_1,t_1)\rangle$$
$$=-i\int dt_1\langle\Phi_{\vec{p}}(\vec{r}_1,t_1)|\left(-\vec{A}(t_1)\cdot\vec{p}\right)|\phi_i(\vec{r}_1,t_1)\rangle$$
$$=i\sum_{n=n_0}^{\infty}\int dt_1A_0e^{i(p^2/2+U_p+I_p-n\omega)}J_n\left(\frac{\vec{p}\cdot\vec{E}_0}{\omega^2},-\frac{z}{2}\right)$$
$$\times\langle\phi_{\vec{p}}(\vec{r}_1)|(\vec{e}_z\cdot\vec{p})|\phi_i(\vec{r}_1)\rangle$$
$$=2\pi i\sum_{n=n_0}^{\infty}A_0J_n\left(\frac{\vec{p}\cdot\vec{E}_0}{\omega^2},-\frac{z}{2}\right)\langle\phi_{\vec{p}}(\vec{r}_1)|(\vec{e}_z\cdot\vec{p})|\phi_i(\vec{r}_1)\rangle$$
$$\times\delta\left(p^2/2+U_p+I_p-n\omega\right), \tag{9}$$

where,$n_0=[(U_P+I_P)/\omega]$ is the minimum necessary number of absorbed photons required for ionization, UP is the electron's ponderomotive energy and IP is the ionization energy. $\vec{E}_0=\vec{A}_0\omega$, $J_n\left(\vec{p}\cdot\vec{E}_0/\omega^2,-z/2\right)$ is the generalized Bessel function of three arguments [4, 14], and $z=U_P/\omega$. $|\phi_i(\vec{r}_1)\rangle$ is initial ground state of the hydrogen atom. And the remaining matrix element:

$$\langle\phi_{\vec{p}}(\vec{r}_1)|(\vec{e}_z\cdot\vec{p})|\phi_i(\vec{r}_1)\rangle$$
$$=\frac{1}{\sqrt{2\pi}}e^{\pi/2p}\Gamma(1-i/p)$$
$$\times\int e^{-i\vec{p}\cdot r_1}F_1\left(-i/p,1,-i(pr+\vec{p}\cdot\vec{r})\right)(\vec{e}_z\cdot\vec{p})e^{-r}d^3r$$
$$=4\sqrt{2}e^{\pi/2p}\Gamma(1-i/p)(\vec{e}_z\cdot\vec{p})(1+i/p)\frac{(1-ip)^{-2-i/p}}{(1+ip)^{-2-i/p}} \tag{10}$$

Next, we calculate the second term of the Coulomb-Volkov S-matrix series $|S_{fi}^{(2)}|^2$. The hydrogen atom only has a long-range Coulomb potential. So

$$S_{fi}^{(2)}=\frac{A_0}{(2\pi)^{4.5}}\int_{-\infty}^{\infty}dt_2\int_{-\infty}^{t_2}dt_1\int d\vec{k}\left\langle\phi_{\vec{p}}(\vec{r}_2)\right|\left(\vec{e}_z\cdot(\hat{p}-\hat{h})\right)\left|\vec{k}\right\rangle$$
$$\times\langle\vec{k}|\left(-\vec{A}(t_1)\cdot\vec{p}+\vec{A}^2(t_1)/2\right)|\phi_i(\vec{r}_1)\rangle$$
$$\times e^{i\int_{-\infty}^{t_2}[(\vec{p}-\vec{A}(\tau)^2)/2]d\tau}e^{-i\int_{-\infty}^{t_2}[(\vec{k}-\vec{A}(\tau)^2)/2]d\tau}$$
$$\times e^{i\int_{-\infty}^{t_1}[(\vec{k}-\vec{A}(\tau)^2)/2]d\tau+iI_pt_1}$$
$$=\frac{A_0}{(2\pi)^{4.5}}\sum_N\sum_{n=n_0}^{\infty}\int dt_2\int d\vec{k}\frac{e^{i(p^2/2-\varepsilon_N)}}{\vec{k}^2/2-\varepsilon_n}\left(U_p-n\omega\right)$$
$$\times J_N\left(\frac{\vec{q}\cdot\vec{E}_0}{\omega^2}\right)J_n\left(\frac{\vec{k}\cdot\vec{E}_0}{\omega^2},-\frac{z}{2}\right)\frac{8/\sqrt{\pi}}{(1+k^2)^2}$$
$$\times\langle\phi_{\vec{p}}(\vec{r}_2)|\vec{e}_z\cdot(\hat{p}-\hat{h})|\vec{k}\rangle \tag{11}$$

where, $\varepsilon_n=n\omega-U_p-I_p$ and $\varepsilon_N=N\omega-U_p-I_p$. **k** is the intermediate momentum, and **q** = **p–k** is the remaining matrix element.

In the current context, from Eq.(12), the transition amplitude is logarithmically divergent for forward scattering, i.e., p = k. For the Coulomb potential, numerical evaluation indicates that for low momentum, $|S_{fi}^{(2)}|^2$ is larger than $|S_{fi}^{(1)}|^2$. Here, in this paper, we focus on the second order term $|S_{fi}^{(2)}|^2$ and ignore the higher-order terms. Basing on the fact of that the ground state decays due to the action of the laser field, we add a decay factor into the phase of the S-matrix element [21] by replacing $I_p t_1$ by $I_p t_1 - i\Lambda t_1/2$. The deplete rate of the ground state gives a finite width to the photoelectron momentum. Then the energy denominator in Eq.(11) is replaced by $\vec{k}^2/2 - \varepsilon_n - i\Lambda/2$.

$$\left\langle \phi_{\vec{p}}(\vec{r}_2)\right| \left(\vec{e}_z \cdot (\hat{p} - \hat{h})\right) \left|\vec{k}\right\rangle$$
$$= 8\sqrt{\pi} e^{\pi/2p} \Gamma(1 - i/p)$$
$$\times \int e^{-i(\vec{p}-\vec{k})\cdot\vec{r}_1} \cdot F_1\left(-i/p, 1, -i(pr + \vec{p}\cdot\vec{r})\right) \left(\vec{e}_z \cdot (\hat{p} - \hat{h})\right) d^3r$$
$$= 8\sqrt{\pi} e^{\pi/2p} \Gamma(1 - i/p) \frac{(\vec{e}_z\cdot\vec{q})}{q^2(q^2-2\vec{q}\cdot\vec{p})} \left(\frac{q^2}{q^2-2\vec{q}\cdot\vec{p}}\right)^{i/p} \quad (12)$$

For given laser parameters, the deplete rate of the ground state Λ is well-defined. We can employ the Coulomb-corrected SFA [14], which is known to be quantitatively agreement with ab initio calculated rates [26]. Besides, we also compare the results of Coulomb-corrected SFA with those of ac-ADK formula [27] in sec.4. It is worthy of noting that the pole approximation [8, 23] cannot be performed when we evaluate integrals over intermediate momentum k.

As the typical short-range potential, i.e., Yukawa potential e^{-kr}/r is widely used [23, 28]. We also calculate the second order term of Coulomb-Volkov S-matrix series that includes Yukawa potential only. Thus, Eq. (12) becomes [29]

$$\left\langle \phi_{\vec{p}}(\vec{r}_2)\right| e^{-kr}/r \left|\vec{k}\right\rangle$$
$$= 2\sqrt{\pi} e^{\pi/2p} \Gamma(1 - i/p)$$
$$\times \int e^{-i(\vec{p}-\vec{k})\cdot\vec{r}_1} F_1\left(-i/p, 1, -i(pr + \vec{p}\cdot\vec{r})\right) e^{-kr}/r \, d^3r$$
$$= \frac{2\sqrt{\pi}}{\alpha} e^{\pi/2p} \Gamma(1 - i/p) \left(\frac{\gamma+\delta}{\gamma}\right)^{-i/p} \left(1 - \frac{\alpha\delta-\beta\gamma}{\alpha(\gamma+\delta)}\right)^{-i/p} \quad (13)$$

where $\alpha = [(\vec{p}-\vec{k})^2 + \kappa^2]/2$, $\beta = \vec{p}\cdot(\vec{k}-\vec{p})^2 + i\kappa p$, $\gamma = \vec{k}\cdot(\vec{k}-\vec{p}) + i\kappa k - \alpha, \delta = pk - \vec{p}\cdot\vec{k} - \beta$, while other parts of Eq.(11) are unchanged. Clearly, for Yukawa potential we do not have the Coulomb singularity due to the existence of the screening parameter κ.

4. Results and discussion

In this section, we calculate the first two order terms of Coulomb-Volkov S-matrix series, i.e., $|S_{fi}^{(1)}|^2$ and $|S_{fi}^{(2)}|^2$. Before, we firstly compare the results of Coulomb-corrected SFA with those of ac-ADK formula [27]. Figure 1 presents our numerical results for the deplete rate of the ground state of the hydrogen atom in the 800 nm laser field. We compare the ionization rate (1) calculated by the first term of the Coulomb-Volkov S-matrix series numerically, as described in sec.3, with the rates obtained by the Coulomb-correction SFA and ac-ADK theory. It can be found that the Coulomb-correction SFA is one to two orders of magnitude higher than the $|S_{fi}^{(1)}|^2$, in the intensity range from 5×10^{13} W/cm^2 to 4×10^{14} W/cm^2. The Coulomb-correction SFA is about one order higher than the ac-ADK theory in the low intensity regime and the Coulomb-Volkov S-matrix series numerically, as described in sec.3, with the rates obtained by the Coulomb-correction SFA and ac-ADK theory. It can be found that the Coulomb-correction SFA is one to two orders of magnitude higher than the $|S_{fi}^{(1)}|^2$, in the intensity range from 5×10^{13} W/cm^2 to 4×10^{14} W/cm^2. The Coulomb-correction SFA is about one order higher than the ac-ADK theory in the low intensity regime and the difference between them becomes inconspicuous as the intensity increases. As we all know, ac-ADK formula can successfully describe the ionization rate in tunneling regime where $\gamma = \sqrt{I_p/2U_p}$. In order to illuminate the role of deplete rate of the ground state in the calculation of $|S_{fi}^{(2)}|^2$, we employ the two different deplete rates obtained from Coulomb-correction SFA and ac-ADK formula.

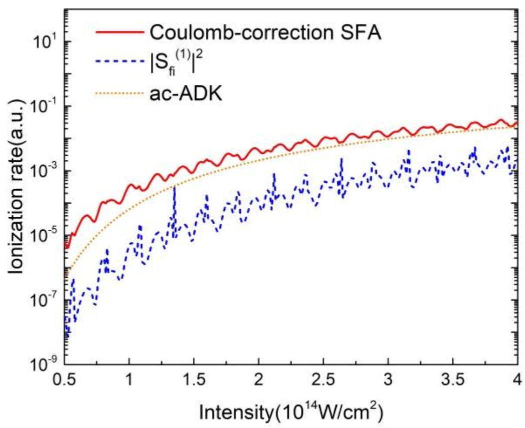

Figure 1: (colour online). Calculated ionization rate for the ground state of the hydrogen atom in the 800 nm laser field by different methods. The red line denotes Coulomb-correction SFA [14]. The blue dashed line denotes the first term of Coulomb-Volkov S-matrix series and the orange short dot line denotes result of ac-ADK formula [27].

Figure 2 demonstrates how the spectrum of photoelectron changes with laser intensities. The deplete rate of the ground state is obtained from Coulomb-correction SFA. We can see that the low-energy part of the spectrum consists of both $|S_{fi}^{(1)}|^2$ and $|S_{fi}^{(2)}|^2$, and has a cutoff near $2Up$. However, the high-energy part (>$2Up$) consists fully of $|S_{fi}^{(2)}|^2$, and shows a plateau with a cutoff near 10Up which becomes higher and wider with increasing intensity. The difference between $|S_{fi}^{(1)}|^2$ and $|S_{fi}^{(2)}|^2$ diminishes in regime of 0∼$2Up$ as the laser intensity increases. This is in good agreement with Coulomb-correction SFA, where the Coulomb-correction factor is inversely proportional to the laser intensity (see Eq. (7) in ref [14]).

For the low-energy part of the spectrum below 2Up, as we can see from Fig. 2, the contribution from $|S_{fi}^{(2)}|^2$ is larger than $|S_{fi}^{(1)}|^2$ by two to four orders of magnitude. It can be expected that $|S_{fi}^{(2)}|^2$ will be even larger than $|S_{fi}^{(1)}|^2$. Nevertheless, we do not know that the complete Coulomb-Volkov S-matrix series of ATI are convergent or do not consider the field-free case of Coulomb scattering [21]. This is still an open question to be investigated in future work.

The reason why the contribution coming from $|S_{fi}^{(2)}|^2$ is larger than $|S_{fi}^{(1)}|^2$ originates from Coulomb scattering. It can be unambiguously showed that, in the Eq. (12), when the final momentum **p** of the ionized electron approaches the intermediate momentum **k**, the denominator increases dramatically. Since

$\langle\vec{p}|V|\vec{k}\rangle \propto 1/(\vec{p}-\vec{k})^2$ for a Coulomb potential of the type $V_C(r) = -1/r$ the rescattering matrix element is maximal for forward scattering.

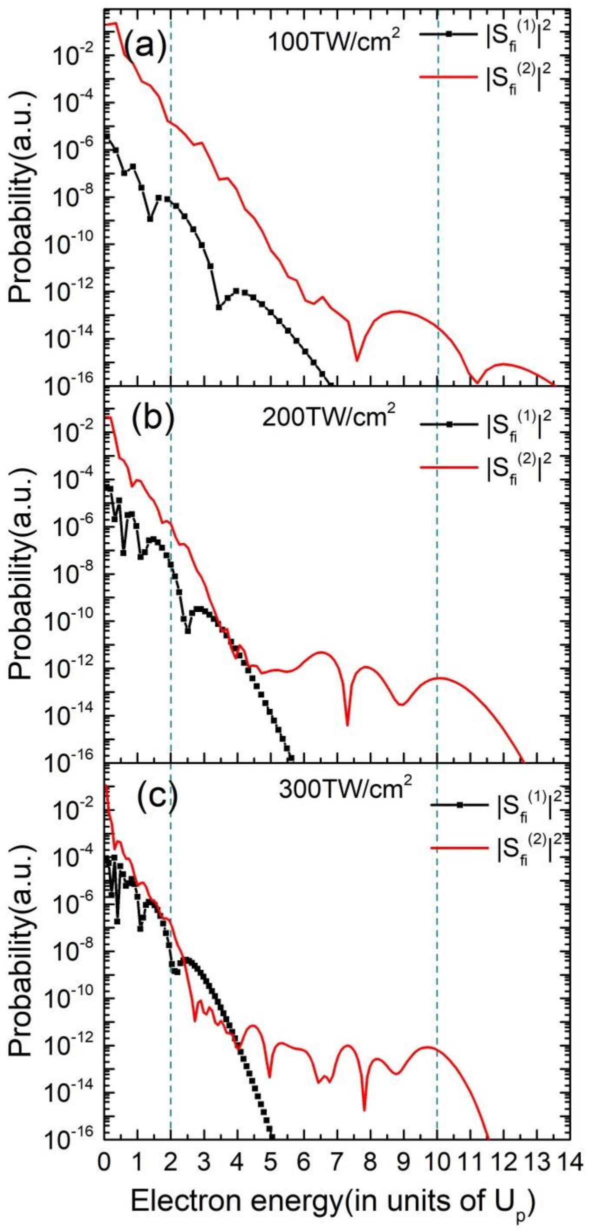

Figure 2: (color online).Photoelectron energy spectra in the laser polarization direction calculated by Eq.(9) and Eq.(11) with three different laser intensities (a):100TW/cm^2, (b):200TW/cm^2, (c):300TW/cm^2 for the hydrogen atom. The wavelength of laser is 800 nm. The corresponding Keldysh parameters γ are 1.06 (a), 0.75 (b) and 0.62 (c).

In order to verify the reliability of this theory, in figure 3, we compare the results obtained using the $|S_{fi}^{(2)}|^2$ with only Coulomb potential with the results obtained with Yukawa potential with different parameters. In the case of Yukawa potential (short-range potential), we do not have the Coulomb singularity, nevertheless, we observed a very high yield at low energy regime when the potential parameter κ is very small. However, no matter how the parameter κ changes, the high energy plateau remains unchanged. This is not difficult to be understood since the smaller the κ, the closer the Yukawa potential approaches the Coulomb potential and the high-energy plateau comes from backscattering of the photoelectron upon the ionic potential which mainly depends on the short-range part of the potential where both the Yukawa and Coulomb potentials are nearly the same.

From the Figure 1, we can see that in contrast to $|S_{fi}^{(1)}|^2$, the deplete rate of the ground state obtained using ac-ADK formula is closer to that obtained using the Coulomb-correction SFA in the laser intensities range from 100 TW/cm^2 to 300 TW/cm^2. Therefore, for the sake of illustrating the influence of the deplete rate of the ground state Λ on the calculation of $|S_{fi}^{(2)}|^2$, we compare $|S_{fi}^{(2)}|^2$ obtained by employing different deplete rates (three kinds of methods) in Figure 4.

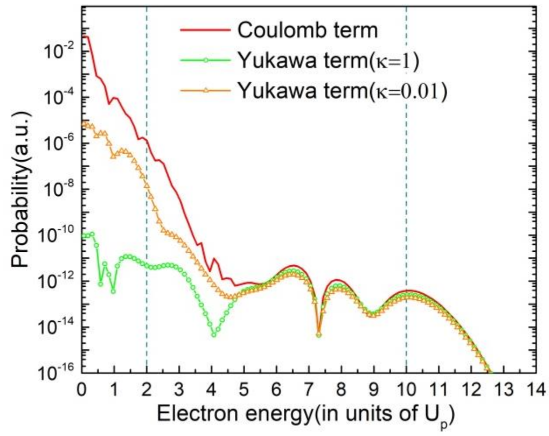

Figure 3: (color online). Comparison of the photoelectron spectra calculated by $|S_{fi}^{(2)}|^2$ with only Coulomb potential with the results of Yukawa potential with different screen parameters (the orange line for κ =0.01 and the green line for κ = 1). The electron emission angle is 0^0, and the wavelength and intensity of the linearly polarized laser field are 800 nm and 200 TW/cm^2, respectively.

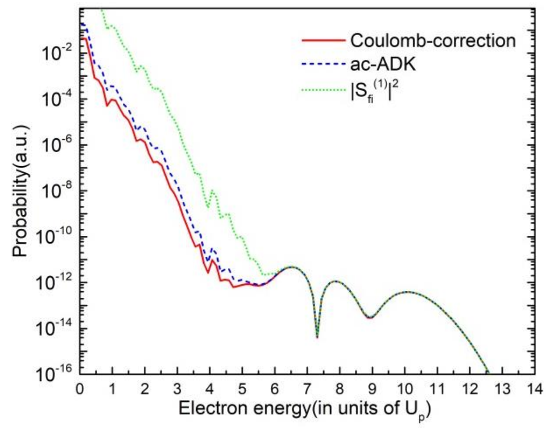

Figure 4: Comparison of the $|S_{fi}^{(2)}|^2$with different depletion rate Λ obtained by different methods (the red line for Coulomb-correction SFA, the green dashed line for ac-ADK formula and the olive dot line for $|S_{fi}^{(1)}|^2$in our calculation). The electron emission angle is 0^0 and the wavelength and intensity of the linearly polarized laser field are 800 nm and 200 TW/cm^2, respectively.

The results presented in Figure. 4 show that the low-energy part of the spectrum becomes more and more lower as the deplete rate increases, while the high-energy plateau almost remains unchanged. As a result, we find that the deplete rate of the ground state added into the phase of the S-matrix element has a significant influence on the magnitude of the low-energy regime (0 ~ $4U_p$), however, makes no difference on the high-energy plateau (6 ~ $10U_p$).

5. Conclusions

We have presented the energy spectra of photoelectron in ATI and HATI process by calculating the first two order terms of the complete Coulomb-Volkov S-matrix series. Within the complete Coulomb-Volkov S-matrix series, effect of the ionic long-range Coulomb potential on the ionized electron is taken into account beyond the first Born approximation with Coulomb-Volkov state. Our analysis shows that high-order terms (starting from the second term) in the complete Coulomb-Volkov S-matrix series still encounter divergence problem due to forward Coulomb scattering amplitude. This divergence can be eliminated by adding a decay factor into the phase of the S-matrix element due to depletion of the atomic ground state without introduction of a suitable screening constant. Calculation shows that the photoelectron energy spectra can be well described by the first two order terms of the complete Coulomb-Volkov S-matrix series. In the low-energy regime below $2U_p$, the contribution from $|S_{fi}^{(2)}|^2$ is larger than $|S_{fi}^{(1)}|^2$ by two to four orders of magnitude. Our results indicate that this distinction is related to the influence of forward-scattered-electron amplitude.

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