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AN OPTIMAL METHOD FOR ADJUSTING THE CENTERING PARAMETER IN THE WIDE-NEIGHBORHOOD PRIMAL-DUAL INTERIOR-POINT ALGORITHM FOR LINEAR PROGRAMMING [*1)]

Wen-bao Ai
(*Department of Mathematics, Beijing University of Posts and Telecommunications, Beijing 100876, China*)

Abstract

In this paper we present a dynamic optimal method for adjusting the centering parameter in the wide-neighborhood primal-dual interior-point algorithms for linear programming, while the centering parameter is generally a constant in the classical wide-neighborhood primal-dual interior-point algorithms. The computational results show that the new method is more efficient.

Mathematics subject classification: 65H10.
Key words: Linear programming, Primal-dual interior point methods, Wide-neighbourhood methods.

1. Introduction

The primal problem and the dual problem we are concerned with are

(LP) $\min c^T x$, s.t. $Ax = b,\ x \geq 0$,

(LD) $\max b^T y$, s.t. $A^T y + s = c,\ s \geq 0$,

where $A \in R^{m\times n}$, $c, x, s \in R^n$ and $y, b \in R^m$. Let

$$F_{++} = \{(x,s) \in R^{2n} : Ax = b,\ A^T y + s = c \ \text{ for some } y \in R^m,\ (x,s) > 0\}.$$

We say that the point (x,s) is an interior feasible pair if $(x,s) \in F_{++}$. We also use the notation $w := (x,s)$.

Let us look back the iteration process of the primal-dual interior point algorithms: Assume that (x,s) is a current iterate interior feasible pair. Let X and S denote the diagonal matrix obtained from the vectors x and s respectively, i.e. $X = diag(x)$ and $S = diag(s)$. Then, the search direction of the typical primal-dual interior point methods is obtained by solving the following linear equations system:

$$\begin{array}{lcl} A\Delta x & = & 0, \\ A^T\Delta y + \Delta s & = & 0, \\ S\Delta x + X\Delta s & = & \gamma\mu e - Xs, \end{array} \tag{1.1}$$

where $\mu = x^T s/n$ and $\gamma \in [0,1]$ is so-called centering parameter. The solution of (1.1) is denoted by $\Delta w(\gamma) = (\Delta x(\gamma), \Delta s(\gamma))$. Then the next iteration pair $w(\gamma,\theta) = (x(\gamma,\theta), s(\gamma,\theta))$ is obtained by setting

$$x(\gamma,\theta) = x + \theta\Delta x(\gamma),$$
$$s(\gamma,\theta) = s + \theta\Delta s(\gamma),$$

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or in a more compact notation

$$w(\gamma,\theta) = w + \theta\Delta w(\gamma),$$

where $\theta \in (0,+\infty)$ is chosen so that $w(\gamma,\theta) \geq 0$ or $w(\gamma,\theta)$ belongs to some neighborhood of the centering path. Two choices of γ have special interest:

(i) $\gamma = 0$: the affine scaling direction $\Delta w^a = (\Delta x^a, \Delta s^a)$;

(ii) $\gamma = 1$: the centering scaling direction $\Delta w^c = (\Delta x^c, \Delta s^c)$.

By superposition, the search direction (1.1) satisfies

$$\Delta w(\gamma) = \gamma \Delta w^c + (1-\gamma)\Delta w^a. \tag{1.2}$$

and the resulting point will be

$$w(\gamma,\theta) = w + \theta\gamma\Delta w^c + \theta(1-\gamma)\Delta w^a. \tag{1.3}$$

Obviosly, the value of γ plays an important role in the primal-dual interior point algorithms. Generally, the typical choices of the centering parameter γ are as follows:

(1) In narrow neighborhood path-following methods([8, 4, 1]), γ is the smallest value such that

$$\|X(\gamma,1)s(\gamma,1) - \mu(\gamma,1)e\| \leq 0.25\mu(\gamma,1),$$

where $\mu(\gamma,1) = x(\gamma,1)^T s(\gamma,1)/n$.

(2) In narrow neighborhood predictor-corrector methods([7]), $\gamma = 0$ and $\gamma = 1$ respectively.

(3) In wide neighborhood methods ([3, 2, 6, 9]), γ is often a small positive constant between 0.001 and 0.005.

We have to remark that, although the methods in (1) and (2) have the best iteration bound $O(\sqrt{n})$ theoretically, they are less practical; and although theoretical iteration bound of the methods in wide neighborhoods (3) is not best (only $O(n)$), they are more practical because they allow long steps, which is a prerequisite for practical efficiency. Notice that γ is a constant for all iterations in wide neighborhood methods, which is not the best choice obviously. It is surely true that practical efficiency will be better if γ can be adaptively adjusted according to the current iteration interior pair. This is really purpose of the paper. In section 2, we introduce a dynamic optimal adjusting method of γ according to the current iteration interior pair. In section 3, we give our algorithms and its convergence proof. Finally, in section 4, we show our computational results.

In this paper, we often use the following conclusion that is well-known and easily proved:

Conclusion A. *For any $\Delta x \in R^n$ and $\Delta s \in R^n$ that satisfy $A\Delta x = 0$ and $A^T\Delta y + \Delta s = 0$, there is*

$$\Delta x^T \Delta s = 0.$$

We also need the terminology $N_\infty^-(\eta)$ from Mizuno, Todd and Ye ([7]), which is

$$N_\infty^-(\eta) := \{(x,s) \mid (x,s) \in F_{++}, Xs \geq (1-\eta)\mu e\},$$

where $\eta \in (0,1)$ and $\mu = x^T s/n$.

2. An Optimal Method for Adjusting γ

Suppose that (x,s) is a current iterate interior feasible pair. We choose a centering parameter γ such that the duality gap of the new iteration pair $w(\gamma)$ would minimized, i.e. γ is a solution of the following optimization problem:

$$\min\ x(\gamma,\theta)^T s(\gamma,\theta) \qquad \text{s.t.}\ \ x(\gamma,\theta) \geq 0 \ \text{ and } \ s(\gamma,\theta) \geq 0. \tag{2.1}$$

From (1.3) and Conclusion A, the above problem is changed into the following:

$$\begin{array}{ll} & \min\ (1-\theta(1-\gamma))\, x^T s \\ s.t. & x+\theta\gamma\Delta x^c+\theta(1-\gamma)\Delta x^a \geq 0, \\ & s+\theta\gamma\Delta s^c+\theta(1-\gamma)\Delta s^a \geq 0. \end{array} \tag{2.2}$$

(2.2) is a nolinear programming problem. If we set $\sigma := \theta\gamma$ and $t := \theta(1-\gamma)$, the problem (2.2) is equivalent to the following linear programming problem:

$$\begin{array}{ll} & \max\ t \\ s.t. & x+\sigma\Delta x^c+t\Delta x^a \geq 0, \\ & s+\sigma\Delta s^c+t\Delta s^a \geq 0. \end{array} \tag{2.3}$$

Now, we try to solve the above problem. For the convenience in notation, we set

$$\left.\begin{array}{l} x_{n+i} := s_i\ , \\ \Delta x^c_{n+i} := \Delta s^c_i\ , \\ \Delta x^a_{n+i} := \Delta s^a_i\ , \end{array}\right. \qquad \text{for } i=1,2,\cdots,n. \tag{2.4}$$

and

$$\begin{array}{l} N_1=\{i:\Delta x^c>0, i=1,2,\cdots,2n\}, \\ N_2=\{i:\Delta x^c<0, i=1,2,\cdots,2n\}, \\ N_3=\{i:\Delta x^c=0, i=1,2,\cdots,2n\}. \end{array}$$

The constraints of (2.3) are equivalent to the following:

$$-\tfrac{x_i}{\Delta x^c_i}-t\tfrac{\Delta x^a_i}{\Delta x^c_i} \leq \sigma \leq -\tfrac{x_j}{\Delta x^c_j}-t\tfrac{\Delta x^a_j}{\Delta x^c_j}, \quad \text{for all } i\in N_1 \text{ and } j\in N_2, \tag{2.5}$$

$$x_i+t\Delta x^a_i \geq 0, \qquad \text{for all } i\in N_3. \tag{2.6}$$

There exists such a σ that satisfies (2.5), if and only if t satisfies the following inequalities:

$$-\frac{x_i}{\Delta x^c_i}-t\frac{\Delta x^a_i}{\Delta x^c_i}+\frac{x_j}{\Delta x^c_j}+t\frac{\Delta x^a_j}{\Delta x^c_j} \leq 0, \quad \text{for all } i\in N_1 \text{ and } j\in N_2,$$

i.e.

$$t\left(\tfrac{\Delta x^a_j}{\Delta x^c_j}-\tfrac{\Delta x^a_i}{\Delta x^c_i}\right) \leq \tfrac{x_i}{\Delta x^c_i}-\tfrac{x_j}{\Delta x^c_j}, \qquad \text{for all } i\in N_1 \text{ and } j\in N_2. \tag{2.7}$$

Note that

$$\frac{x_i}{\Delta x^c_i}-\frac{x_j}{\Delta x^c_j}>0, \text{ for all } i\in N_1 \text{ and } j\in N_2.$$

So, for some $i\in N_1$ and $j\in N_2$, if there is

$$\frac{\Delta x^a_j}{\Delta x^c_j}-\frac{\Delta x^a_i}{\Delta x^c_i} \leq 0,$$

then the inequality (2.7) is true for all $t \geq 0$. We set

$$\begin{array}{l} t_1=\min\left\{\frac{\frac{x_i}{\Delta x^c_i}-\frac{x_j}{\Delta x^c_j}}{\frac{\Delta x^a_j}{\Delta x^c_j}-\frac{\Delta x^a_i}{\Delta x^c_i}} \middle| \frac{\Delta x^a_j}{\Delta x^c_j}-\frac{\Delta x^a_i}{\Delta x^c_i}>0, i\in N_1 \text{ and } j\in N_2\right\}, \\ t_2=\min\left\{-\frac{x_i}{\Delta x^a_i} \middle| \Delta x^a_i<0, i\in N_3\right\}, \\ t^*=\min\{t_1,t_2\}, \end{array} \tag{2.8}$$

and

$$\begin{aligned}\sigma_1 &= \max\left\{-\tfrac{x_i}{\Delta x_i^c} - t^* \tfrac{\Delta x_i^a}{\Delta x_i^c} | i \in N_1\right\},\\ \sigma_2 &= \min\left\{-\tfrac{x_i}{\Delta x_i^c} - t^* \tfrac{\Delta x_i^a}{\Delta x_i^c} | i \in N_2\right\}.\end{aligned} \tag{2.9}$$

Notice that $\sigma_1 \le \sigma_2$ because of the choice of t^*. Then, any point (t^*, σ^*) that satisfies the following inequalities

$$\sigma_1 \le \sigma^* \le \sigma_2, \tag{2.10}$$

is a solution of (2.3). So, we obtain the solutions of problem (2.1):

$$\theta^* = \sigma^* + t^*, \quad \gamma^* = \sigma^*/(\sigma^* + t^*). \tag{2.11}$$

Lemma 2.1. *For any σ and t, if*

$$x + \sigma\Delta x^c + t\Delta x^a \ge 0, \tag{2.12}$$

$$s + \sigma\Delta s^c + t\Delta s^a \ge 0, \tag{2.13}$$

then

$$\sigma(\mu e - Xs) + (2 - t)Xs \ge 0. \tag{2.14}$$

Proof. Multiplying the both sides of (2.12) and (2.13) by S and X from left respectively, we obtain

$$Sx + \sigma S\Delta x^c + tS\Delta x^a \ge 0, \tag{2.15}$$

$$Xs + \sigma X\Delta s^c + tX\Delta s^a \ge 0. \tag{2.16}$$

By adding the both sides of (2.15) to the both corresponding sides of (2.16) we have

$$2Xs + \sigma(S\Delta x^c + X\Delta s^c) + t(S\Delta x^a + X\Delta s^a) \ge 0. \tag{2.17}$$

From (1.1), the inequalities (2.17) is equivalent to the following

$$2Xs + \sigma(\mu e - Xs) - tXs \ge 0.$$

The proof has ended.

Lemma 2.2. *Suppose that $(x, s) \in N_\infty^-(\eta)$ and let t^* be defined by (2.8). Then*

$$\underline{t} \le t^* \le 1,$$

where

$$\underline{t} = \frac{1}{\sqrt{1 + n/(1-\eta)}}.$$

Proof. Notice that

$$x + \sigma^*\Delta x^c + t^*\Delta x^a \ge 0,$$

$$s + \sigma^*\Delta s^c + t^*\Delta s^a \ge 0,$$

and from Conclusion A

$$(\sigma^*\Delta x^c + t^*\Delta x^a)^T(\sigma^*\Delta s^c + t^*\Delta s^a) = 0,$$

as well as from (1.1) we obtain

$$x^T\Delta s^c + s^T\Delta x^c = n\mu - x^T s = 0,$$

and

$$x^T \Delta s^a + s^T \Delta x^a = -x^T s.$$

Then

$$\begin{aligned}&(x + \sigma^* \Delta x^c + t^* \Delta x^a)^T (s + \sigma^* \Delta s^c + t^* \Delta s^a)\\ &= x^T s + \sigma^* (x^T \Delta s^c + s^T \Delta x^c) + t^* (x^T \Delta s^a + s^T \Delta x^a)\\ &= (1 - t^*) x^T s \geq 0,\end{aligned}$$

which implies $t^* \leq 1$.

The rest is to prove the other inequality $t^* \geq \underline{t}$. From (1.1), Lemma 4.14 (iii) and Lemma 4.15 (i) of ([9]), we have for each $t \in [0, \underline{t}]$

$$\begin{aligned}&(X + t\Delta X^a)(s + t\Delta s^a)\\ &= (1-t)Xs + t^2 \Delta X^a \Delta s^a\\ &\geq (1-t)Xs - t^2 \left\|\Delta X^a \Delta s^a\right\|_\infty e\\ &\geq (1-t)(1-\eta)\mu e - \tfrac{1}{4} t^2 n \mu e\\ &\geq \left((1-\underline{t})(1-\eta) - \tfrac{1}{4}\underline{t}^2 n\right) \mu e\\ &\geq 0.\end{aligned}$$

Therefore from continuity

$$\begin{aligned}x + \underline{t}\Delta x^a \geq 0,\\ s + \underline{t}\Delta s^a \geq 0.\end{aligned} \tag{2.18}$$

(2.18) show that $\sigma = 0$ and $t = \underline{t}$ is a feasible point of problem (2.3), which implies that $\underline{t} \leq t^*$. The proof has ended.

3. Algorithm

Now we can give our primal-dual interior point algorithm.

Algorithm 1.

Step 0: Input $\varepsilon > 0$. Let $\eta \in (0,1)$ and $\underline{\gamma} \in (0,1)$ with $\underline{\gamma} \leq 2(1-\eta)$. Given an initial point $(x^0, s^0) \in N_\infty^-(\eta)$. Compute $\mu_0 = (x^0)^T s^0 / n$. Set $k := 0$.

Step 1: If $(x^k)^T s^k \leq \varepsilon$ then stop.

Step 2: Set $(x, s) = (x^k, s^k)$ and compute $(\Delta x^a, \Delta s^a)$ and $(\Delta x^c, \Delta s^c)$ from (1.1) for $\gamma = 0$ and $\gamma = 1$ respectively. Compute $t^* and\ \sigma^* := \sigma_2$ from (2.8)-(2.9). If $\sigma^* + t^* \neq 0$, set $\gamma^* = \sigma^* / (\sigma^* + t^*)$ and

$$\gamma^k = \begin{cases} \underline{\gamma}, & \text{if } \sigma^* + t^* = 0 \text{ or } \gamma^* \leq \underline{\gamma},\\ \gamma^*, & \text{otherwise.}\end{cases}$$

Step 3: Compute the largest $\overline{\theta}$ such that

$$\left(x(\gamma^k, \theta), s(\gamma^k, \theta)\right) \in N_\infty^-(\eta) \quad \text{for all } \theta \in [0, \overline{\theta}];$$

set $(x^{k+1}, s^{k+1}) = \left(x(\gamma^k, \overline{\theta}), s(\gamma^k, \overline{\theta})\right)$.

Step 4: Let $k := k+1$, return to Step 1 and repeated.

Lemma 3.1. *In Algorithm 1, if $\gamma^k = \gamma^*$, then*

$$\mu_{k+1} \leq \left(1 - \underline{\gamma}\eta {t^*}^2 / 2\right) \mu_k .$$

Proof. Set

$$\begin{array}{lcl} \mu & = & \mu^k\,, \\ \delta: & = & \theta(1-\gamma^k)/t^*\,, \\ \overline{\delta}: & = & \overline{\theta}(1-\gamma^k)/t^*\,, \\ p: & = & (\sigma^*\Delta X^c + t^*\Delta X^a)(\sigma^*\Delta s^c + t^*\Delta s^a), \\ q: & = & (X+\sigma^*\Delta X^c + t^*\Delta X^a)(s+\sigma^*\Delta s^c + t^*\Delta s^a), \\ \mu(\theta): & = & x(\gamma^k,\theta)^T s(\gamma^k,\theta)/n\,. \end{array}$$

The following relations (a1), (a2), (a3) and (a4) are obviously true:

(a1) $\theta \in [0,\overline{\theta}]$ if and only if $\delta \in [0,\overline{\delta}]$. Besides, $\sigma^* = \gamma^* t^*/(1-\gamma^*) \geq \underline{\gamma} t^*$ and $\delta\sigma^* = \theta\gamma^*$.

(a2) $q \geq 0$.

(a3) $p = q - (1-t^*)Xs - \sigma^*(\mu e - Xs)$.

(a4) $\mu(\theta) = (1-\delta t^*)\mu$.

Thus from (a2) and (a3)

$$\begin{array}{l} X(\gamma^k,\theta)s(\gamma^k,\theta) \\ = (X+\delta\sigma^*\Delta X^c + \delta t^*\Delta X^a)(s+\delta\sigma^*\Delta s^c + \delta t^*\Delta s^a) \\ = (1-\delta t^*)Xs + \delta\sigma^*(\mu e - Xs) + \delta^2 p \\ = (1-\delta t^*)Xs + \delta\sigma^*(\mu e - Xs) + \delta^2\left(q - (1-t^*)Xs - \sigma^*(\mu e - Xs)\right) \\ = \delta^2 q + (1-\delta t^* - \delta^2 + \delta^2 t^*)Xs + \delta(1-\delta)\sigma^*(\mu e - Xs) \\ \geq (1-\delta t^* - \delta^2 + \delta^2 t^*)Xs + \delta(1-\delta)\sigma^*(\mu e - Xs). \end{array} \tag{3.1}$$

For each $\delta \in [0, \underline{\gamma}\eta t^*/2]$ and each $i \in \{1,2,\cdots,n\}$, we consider the following two cases:

(1) if $x_i s_i \leq (1-\eta/2)\mu$, from (3.1) and (a4) we have

$$\begin{array}{l} x_i(\gamma^k,\theta)s_i(\gamma^k,\theta) \\ \geq (1-\delta t^* - \delta^2 + \delta^2 t^*)(1-\eta)\mu + \frac{1}{2}\delta(1-\delta)\sigma^*\eta\mu \\ \geq (1-\eta)(1-\delta t^*)\mu - \delta^2(1-\eta)\mu + \frac{1}{2}\delta(1-\delta)\underline{\gamma} t^*\eta\mu \\ \geq (1-\eta)\mu(\theta) + \frac{1}{2}\delta\mu(\underline{\gamma} t^*\eta - 2\delta) \\ \geq (1-\eta)\mu(\theta). \end{array} \tag{3.2}$$

(2) otherwise i.e. if $x_i s_i > (1-\eta/2)\mu$, from (3.1), (a4), Lemma 2.1 and Lemma 2.2 we have

$$\begin{array}{l} x_i(\gamma^k,\theta)s_i(\gamma^k,\theta) \\ \geq (1-\delta t^* - \delta^2 + \delta^2 t^*)Xs - \delta(1-\delta)(2-t^*)Xs \\ = (1-\delta)^2 Xs \\ \geq (1-2\delta)(1-\eta/2)\mu \\ = (1-\delta)(1-\eta)\mu + (\eta/2-\delta)\mu \\ \geq (1-\eta)\mu(\theta). \end{array} \tag{3.3}$$

(3.2) and (3.3) imply that $\overline{\delta} \geq \underline{\gamma}\eta t^*/2$. Therefore

$$\mu^{k+1} = \mu(\overline{\theta}) = (1-\overline{\delta}t^*)\mu \leq (1-\overline{\delta}t^{*2}/2)\mu,$$

which show that the conclusion holds. The proof has ended.

Theorem 3.2. *Let $\eta \in (0,1)$ and $\underline{\gamma} \in (0,1)$ be a constant with $\underline{\gamma} \leq 2(1-\eta)$. Then Algorithm 1 will terminate in $O(n\log\left((x^0)^T s^0/\varepsilon\right)$ iterations.*

Proof. Consider the k-th iteration. if $\gamma^k = \underline{\gamma}$, then from the Theorem 4.20 of ([9])

$$\mu_{k+1} \le \left(1 - \frac{4\eta\underline{\gamma}(1-\underline{\gamma})}{n}\right)\mu_k\,; \tag{3.4}$$

Otherwise, $\gamma^k = \gamma^*$. From Lemma 3.1

$$\begin{aligned} &\mu_{k+1} \le \left(1 - \underline{\gamma}\eta {t^*}^2/2\right)\mu_k\,,\\ &\le (1 - \underline{\gamma}\eta \underline{t}^2/2)\mu_k\,,\\ &\le \left(1 - \tfrac{\underline{\gamma}\eta(1-\eta)}{2-2\eta+2n}\right)\mu_k\,, \end{aligned} \tag{3.5}$$

(3.4) and (3.5) yields the results. The proof has ended.

4. Computational Results

We program all algorithms by FORTRAN-77. In actual computation, we get rid of the safeguard $\underline{\gamma}$. Besides, long-step strategy similar to Lustig ([5])(here we choose 0.996) is used in all algorithms except for predictor-corrector algorithm. All algorithms are repeated until the relative duality gap satisfies

$$\frac{x^T s}{1 + |c^T x|} < 10^{-6} \tag{4.1}$$

or the iterations number ≥ 80.

In Table 1, the problems used by us are the well-known Klee-Minty examples as following:

$$\begin{aligned} &\max \sum_{i=1}^{n} x_i\\ s.t.\quad &x_1 \le 2,\\ &x_i + 2\sum_{j=1}^{i-1} x_j \le 2^i, \quad i = 2,3,\cdots,n;\\ &x_1, x_2, \cdots, x_n \ge 0. \end{aligned}$$

The solution and optimal value of the above problems are $x^* = (0,0,\cdots,0,2^n)$ and $f^* = 2^n$ respectively. Their KKT-system can be written as following:

$$\begin{aligned} &(A,I)x = b,\\ &(-I,A^T)s = e,\\ &Xs = 0,\\ &x \ge 0,\ s \ge 0, \end{aligned}$$

where $b = (2, 2^2, \cdots, 2^n) \in R^n$, $x, s \in R^{2n}$, I is n by n unitary matrix and A is also n by n matrix that

$$A = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0\\ 2 & 1 & 0 & \cdots & 0 & 0\\ 2 & 2 & 1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 2 & 2 & 2 & \cdots & 1 & 0\\ 2 & 2 & 2 & \cdots & 2 & 1 \end{pmatrix}$$

The initial point chosen is

$$\left\{\begin{array}{ll} x_i^0 = 1, & s_i^0 = 4^{n-i} + 1,\\ x_{n+i}^0 = 2^i - 2i + 1, & s_{n+i}^0 = 2. \end{array}\right. \qquad i = 1, 2, \cdots, n$$

Examples in the other tables come from Netlib. We use Ye's self-dual model (see Page 159-168 and Page 355 of [9]) in order to obtain an initial point: $x^0 = s^0 = e, \tau^0 = \kappa^0 = 1$.

Table 1: Results on Klee-Minty examples

Problem		$\gamma =$ this paper's		$\gamma = 1/n$	$\gamma = 0.02$	$\gamma = 0.002$	$\gamma = 0$
Row	Col.	Iter.	$F(x^k)$	Iter.	Iter.	Iter.	Iter.
4	8	6	0.15999997E+02	9	7	7	10
5	10	7	0.31999999E+02	9	7	8	12
6	12	7	0.63999996E+02	8	7	8	13
7	14	7	0.12800000E+03	9	7	8	14
8	16	6	0.25599989E+03	8	7	8	16
9	18	7	0.51199999E+03	8	7	8	17
10	20	6	0.10239994E+04	8	7	8	18
11	22	6	0.20479988E+04	8	8	8	19
12	24	6	0.40959977E+04	8	8	8	20
13	26	7	0.81919999E+04	9	8	8	21
14	28	7	0.16384000E+05	9	9	8	22
15	30	7	0.32767999E+05	9	9	8	23
16	32	7	0.65535999E+05	9	10	8	24
17	34	7	0.13107200E+06	9	10	8	25
18	36	7	0.26214398E+06	11	11	8	26
19	38	7	0.52428790E+06	12	10	9	27
20	40	8	0.10485759E+07	12	12	10	28
21	42	8	0.20971511E+07	13	13	10	29
22	44	8	0.41943031E+07	14	14	11	30
23	46	8	0.83886069E+07	14	14	11	31
24	48	8	0.16777212E+08	14	14	12	32
25	50	8	0.33554431E+08	16	16	13	33
26	52	7	0.67108853E+08	16	16	13	34
27	54	7	0.13421772E+09	16	16	13	35
28	56	7	0.26843544E+09	16	16	14	36
29	58	7	0.53687066E+09	17	18	14	37
30	60	8	0.10737418E+10	18	18	15	38
31	62	8	0.21474837E+10	18	18	15	39

Table 2: Results for $\gamma =$ this paper's

Name	Row	Col.	Iter.	Relative Gaps	$F(x^k)$
ADLITTLE	56	138	18	0.44023789E-07	0.22549489E+06
AFIRO	27	51	13	0.28608252E-07	-0.46475313E+03
BLEND	74	114	15	0.28340984E-07	-0.30812150E+02
SCAGR7	129	185	17	0.89301159E-06	-0.23313553E+07
SHARE1B	117	253	41	0.78942865E-08	-0.76589318E+05
SHARE2B	96	162	17	0.93706072E-07	-0.41573223E+03
SC50A	50	78	16	0.10932362E-08	-0.64575077E+02
SC50B	50	78	13	0.16741997E-07	-0.70000000E+02
KB2	52	77	26	0.83410639E-08	-0.17499001E+04

Table 3: Results for $\gamma = 1/n$

Name	Row	Col.	Iter.	Gaps	$F(x^k)$
ADLITTLE	56	138	23	0.71694938E-06	0.22549384E+06
AFIRO	27	51	17	0.32716745E-07	-0.46475313E+03
BLEND	74	114	19	0.38299857E-07	-0.30812149E+02
SCAGR7	129	185	26	0.30087636E-06	-0.23313780E+07
SHARE1B	117	253	*80	0.21619351E-02	-0.76513640E+05
SHARE2B	96	162	22	0.89574785E-06	-0.41573204E+03
SC50A	50	78	23	0.16651119E-09	-0.64575077E+02
SC50B	50	78	21	0.17908760E-09	-0.70000000E+02
KB2	52	77	37	0.10070738E-07	-0.17499001E+04

Table 4: Results for $\gamma = 0.002$

Name	Row	Col.	Iter.	Gaps	$F(x^k)$
ADLITTLE	56	138	24	0.71694938E-06	0.22549384E+06
AFIRO	27	51	18	0.32716745E-07	-0.46475313E+03
BLEND	74	114	19	0.38299857E-07	-0.30812149E+02
SCAGR7	129	185	27	0.30087636E-06	-0.23313780E+07
SHARE1B	117	253	*80	0.21619351E-02	-0.76513640E+05
SHARE2B	96	162	23	0.89574785E-06	-0.41573204E+03
SC50A	50	78	23	0.16651119E-09	-0.64575077E+02
SC50B	50	78	19	0.17908760E-09	-0.70000000E+02
KB2	52	77	38	0.10070738E-07	-0.17499001E+04

Table 5: Results for predictor-corrector method

Name	Row	Col.	Iter.	Gaps	$F(x^k)$
ADLITTLE	56	138	57	0.82767491E-06	0.22549366E+06
AFIRO	27	51	48	0.99578063E-06	-0.46475281E+03
BLEND	74	114	53	0.87439251E-06	-0.30812140E+02
SCAGR7	129	185	68	0.85305164E-07	-0.23313865E+07
SHARE1B	117	253	*80	0.52854784E-02	-0.76425066E+05
SHARE2B	96	162	64	0.86048553E-07	-0.41573223E+03
SC50A	50	78	65	0.93055485E-07	-0.64575074E+02
SC50B	50	78	65	0.80474578E-07	-0.69999998E+02
KB2	52	77	69	0.91977771E-07	-0.17499001E+04

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