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STRUCTURED BACKWARD ERRORS FOR STRUCTURED KKT SYSTEMS [*1)]

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Abstract

In this paper we study structured backward errors for some structured KKT systems. Normwise structured backward errors for structured KKT systems are defined, and computable formulae of the structured backward errors are obtained. Simple numerical examples show that the structured backward errors may be much larger than the unstructured ones in some cases.

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Key words: Structured — KKT system, Structured backward error, Normwise backward error.

1. Introduction

Consider the problem of solving the following structured linear systems

$$\begin{pmatrix} I & A \\ A^T & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \tag{1}$$

for x and y, where $A \in \mathcal{R}^{m\times n}, x, b \in \mathcal{R}^m, y \in \mathcal{R}^n$;

$$\begin{pmatrix} I & A \\ A^T & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \tag{2}$$

for X and Y, where $A \in \mathcal{R}^{m\times n}, B, X \in \mathcal{R}^{m\times r}, Y \in \mathcal{R}^{n\times r}$;

$$\begin{pmatrix} 0 & 0 & B \\ 0 & I & A \\ B^T & A^T & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} d \\ b \\ 0 \end{pmatrix}, \tag{3}$$

and

$$\begin{pmatrix} 0 & 0 & C & B \\ 0 & I & 0 & I \\ C^T & 0 & 0 & 0 \\ B^T & I & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{4}$$

These systems called augmented systems, and they are structured Karush–Kuhn–Tucker (structured — KKT) systems. The structured – KKT systems of (1) – (4) arise in many applications, for example, for the linear least squares problem

$$\text{LS}: \quad \min_{y\in\mathcal{R}^n} \|b - Ay\|_2, \quad A \in \mathcal{R}^{m\times n}, \quad b \in \mathcal{R}^m,$$

let $r = b - Ay$, then the LS minimizer y satisfies the structured — KKT system (1) since this is simply a representation of the normal equations. The structured — KKT systems (2) – (4)

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mainly arise in the linear least squares problem with multiple right-hand sides[19], the linear least squares problem with equality constraint[3,4], and the generalized linear least squares problem[14], respectively. And problems of least squares problems arise in many fields of study.

The structured linear systems (1) – (4) are structured — KKT systems, and they have stronger structure than the KKT systems, so the results of Sun[16] about the KKT systems are invalid to the structured linear systmes (1) – (4).Consequently, the structured optimal backward perturbation analysis of the structured — KKT systems (1) – (4) is worth researching.

Consider the linear system $Ax = b$. Let $\hat{x}$ be a computed solution to the system. In general, there are many perturbations ΔA, and Δb such that $\hat{x}$ is a solution to the perturbed systems $(A + \Delta A)x = b + \Delta b$. It may be asked how close is the nearest system for which $\hat{x}$ is the solution to the original system. There are various approaches to define backward errors (BEs) for measuring the distance between the perturbed systems and the original systems. Finding an explicit expression of a BE may be very useful for testing the stability of practical algorithms. For general linear system, Rigal and Gaches[15] have defined a normwise BE and obtained the explicit expression

$$\tau(\hat{x}) = \frac{\|b - A\hat{x}\|_2}{\sqrt{\|A\|_F^2\|\hat{x}\|_2^2 + \|b\|_2^2}}.$$

However, it is worth pointing out that if the coefficient matrix A has some special structure, and the perturbed matrice $A + \Delta A$ has the same form as A, in which we are interested. The problem of finding an expression of the corresponding BE should be concerned. Generally speaking, Rigal and Gaches' result is a strict lower bound of the structured BE.

In this paper we shall define the structured BEs of Eq. (1)-(4), derive computable formulae of them, and show that if the perturbed matrices have the same form as the coefficient matrix and the unstructured backward error $\tau(\hat{x})$ is small, it does not necessarily follow that $\hat{x}$ solves a nearby structured linear system.

2. Structured Backward Errors for Structured-KKT Systems

Firstly, we investigate the backward errors for structured-KKT systems (1).

Theorem 2.1. *Let $(\hat{x}^T, \hat{y}^T)^T$ with $\hat{y} \neq 0$ be a computed solution of Eq. (1). Define the normwise structured backward error $\eta^{(\theta)}(\hat{x}, \hat{y})$ of the Eq. (1) by*

$$\eta^{(\theta)}(\hat{x}, \hat{y}) = \min_{(\Delta A, \Delta b) \in \mathcal{E}} \|(\Delta A, \theta \Delta b)\|_F, \tag{5}$$

where θ is a positive parameter, and the set $\mathcal{E}$ is defined by

$$\mathcal{E} = \left\{ (\Delta A, \Delta b) : \begin{pmatrix} I & A + \Delta A \\ (A + \Delta A)^T & 0 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} b + \Delta b \\ 0 \end{pmatrix} \right\}. \tag{6}$$

Then

$$\eta^{(\theta)}(\hat{x}, \hat{y}) = \left[\hat{x}^\dagger (AA^T + \theta^2 (\hat{x} - b)(\hat{x} - b)^T)\hat{x} + \tau \|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y} - b)\|_2^2 \right]^{1/2}. \tag{7}$$

The corresponding perturbations of A and b are

$$\begin{aligned} \Delta A^* &= -\hat{x}\hat{x}^\dagger A + \tau (I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y})\hat{y}^T \quad, \\ \Delta b^* &= \hat{x}\hat{x}^\dagger (\hat{x} - b) - \frac{1}{1 + \theta^2 \|\hat{y}\|_2^2} (I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y}) \ . \end{aligned} \tag{8}$$

That is $\eta^{(\theta)}(\hat{x}, \hat{y}) = \|(\Delta A^*, \theta \Delta b^*)\|_F$, where $\tau = \dfrac{\theta^2}{1 + \theta^2 \|\hat{x}\|_2^2}$.

Proof. By Eq. (6), $(\Delta A, \Delta b) \in \mathcal{E}$ if and only if ΔA and Δb satisfy

$$\Delta b = \hat{x} - b + A\hat{y} + \Delta A\hat{y}, \tag{9}$$

$$\hat{x}^T \Delta A = -\hat{x}^T A. \tag{10}$$

From (10),

$$\Delta A = -\hat{x}\hat{x}^\dagger A + (I_m - \hat{x}\hat{x}^\dagger)Z, \quad Z \in \mathcal{R}^{m \times n}. \tag{11}$$

Substituting it into (9), we get

$$\Delta b = (I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \hat{x} - b + (I_m - \hat{x}\hat{x}^\dagger)Z\hat{y}. \tag{12}$$

Combining (11) and (12) with (5) gives

$$\begin{aligned}(\eta^{(\theta)}(\hat{x},\hat{y}))^2 &= \min_{(\Delta A,\Delta b)\in\mathcal{E}} \|(\Delta A, \theta\Delta b)\|_F^2\\ &= \min_{(\Delta A,\Delta b)\in\mathcal{E}} (tr(\Delta A^T\Delta A) + \theta^2\Delta b^T\Delta b)\\ &= \hat{x}^\dagger AA^T\hat{x} + \theta^2(A\hat{y})^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \theta^2(\hat{x}-b)^T(\hat{x}-b)\\ &\quad -2\theta^2 b^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \min_{Z\in\mathcal{R}^{m\times n}} f(Z),\end{aligned} \tag{13}$$

where

$$\begin{aligned}f(Z) &= tr(Z^T(I_m - \hat{x}\hat{x}^\dagger)Z) + \theta^2(Z\hat{y})^T(I_m - \hat{x}\hat{x}^\dagger)Z\hat{y} + 2\theta^2(A\hat{y}-b)^T(I_m - \hat{x}\hat{x}^\dagger)Z\hat{y}\\ &= \|(I_m - \hat{x}\hat{x}^\dagger)Z\|_F^2 + \theta^2\|(I_m - \hat{x}\hat{x}^\dagger)Z\hat{y}\|_2^2 + 2\theta^2(A\hat{y}-b)^T(I_m - \hat{x}\hat{x}^\dagger)Z\hat{y}\\ &= (vecZ)^T[(I_n + \theta^2\hat{y}\hat{y}^T)\otimes(I_m - \hat{x}\hat{x}^\dagger)]vecZ + 2\theta^2[\hat{y}^T\otimes(A\hat{y}-b)^T(I_m - \hat{x}\hat{x}^\dagger)]vecZ.\end{aligned} \tag{14}$$

Let

$$\begin{aligned}&z = vecZ, \quad M = (I_n + \theta^2\hat{y}\hat{y}^T)\otimes(I_m - \hat{x}\hat{x}^\dagger),\\ &D = (I_n + \theta^2\hat{y}\hat{y}^T)\otimes I_m > 0, \quad c = \hat{y}\otimes[(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)].\end{aligned} \tag{15}$$

Then

$$g(z) =: f(Z) = z^TMz + 2\theta^2c^Tz. \tag{16}$$

It is easy to verify that for

$$\bar{z} = -\theta^2D^{-1}c, \tag{17}$$

we have

$$\begin{aligned}&g(z) - g(\bar{z}) = z^TMz + 2\theta^2c^Tz - \bar{z}^TM\bar{z} - 2\theta^2c^T\bar{z}\\ &= (z-\bar{z})^TM(z-\bar{z}) + 2\bar{z}^TM(z-\bar{z}) + 2\theta^2c^T(z-\bar{z}).\end{aligned}$$

From (15), and (16)

$$\bar{z}^TM = -\theta^2c^TD^{-1}M = -\theta^2c^T$$

Consequently,

$$g(z) - g(\bar{z}) = (z-\bar{z})^TM(z-\bar{z}) \geq 0, \quad \forall z\in\mathcal{R}^{mn}.$$

Thus

$$\begin{aligned}&\min_{Z\in\mathcal{R}^{m\times n}} f(Z) = \min_{z\in\mathcal{R}^{mn}} g(z) = g(\bar{z}) = -\theta^4c^TD^{-1}c\\ &= -\theta^4\hat{y}^T(I_n + \theta^2\hat{y}\hat{y}^T)^{-1}\hat{y}\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= -\theta^4\hat{y}^T(I_n - \tau\hat{y}\hat{y}^T)\hat{y}\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= (-\theta^4\|\hat{y}\|_2^2 + \tau\theta^4\|\hat{y}\|_2^4)\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= (\tau-\theta^2)\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\end{aligned} \tag{18}$$

where

$$\tau = \frac{\theta^2}{1+\theta^2\|\hat{x}\|_2^2} \tag{19}$$

Combining (18) with (13), we have

$$\begin{aligned}(\eta^{(\theta)}(\hat{x},\hat{y}))^2 &= \hat{x}^\dagger AA^T\hat{x} + \theta^2(A\hat{y})^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \theta^2(\hat{x}-b)^T(\hat{x}-b)\\ &\quad -2\theta^2b^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + (\tau-\theta^2)\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= \hat{x}^\dagger AA^T\hat{x} + \theta^2(A\hat{y})^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y} - 2\theta^2b^T(I_m - \hat{x}\hat{x}^\dagger)A\hat{y}\\ &\quad +\theta^2b^T(I_m - \hat{x}\hat{x}^\dagger)b + \theta^2(\hat{x}-b)^T\hat{x}\hat{x}^\dagger(\hat{x}-b) + (\tau-\theta^2)\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= \hat{x}^\dagger(AA^T + \theta^2(\hat{x}-b)(\hat{x}-b)^T)\hat{x} + \theta^2\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &\quad +(\tau-\theta^2)\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2\\ &= \hat{x}^\dagger(AA^T + \theta^2(\hat{x}-b)(\hat{x}-b)^T)\hat{x} + \tau\|(I_m - \hat{x}\hat{x}^\dagger)(A\hat{y}-b)\|_2^2.\end{aligned} \tag{20}$$

From (16) and (14), we have

$$\bar{Z} = \tau(I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y})\hat{y}^T. \tag{21}$$

Combining (21), (18) with (9) and (10) gives the minimum perturbations of A and b in normwise

$$\begin{aligned}\Delta A^* &= -\hat{x}\hat{x}^\dagger A + \tau(I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y})\hat{y}^T,\\ \Delta b^* &= (I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \hat{x} - b + \tau\|\hat{y}\|_2^2(I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y})\\ &= (I_m - \hat{x}\hat{x}^\dagger)A\hat{y} + \hat{x} - b + (1 - \frac{1}{1+\theta^2\|\hat{y}\|_2^2})(I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y})\\ &= \hat{x}\hat{x}^\dagger(\hat{x} - b) - \frac{1}{1+\theta^2\|\hat{y}\|_2^2}(I_m - \hat{x}\hat{x}^\dagger)(b - A\hat{y}).\end{aligned}$$

Similar to the proof of theorem 2.1, we can get the structured backward errors of the structured-KKT systems (2)-(4) as follows:

Theorem 2.2. *Let $(\hat{X}^T, \hat{Y}^T)^T$, $(\hat{x}_1^T, \hat{x}_2^T, \hat{x}_3^T)^T$ and $(\tilde{x}_1^T, \tilde{x}_2^T, \tilde{x}_3^T, \tilde{x}_4^T)^T$ be the computed solutions of Eq. (2),(3) and (4) respectively. Define the normwise structured backward errors of the Eq. (2),(3) and (4) as follows:*

$$\eta_F^{(\theta)}(\hat{X}, \hat{Y}) = \min_{(\Delta A, \Delta B)\in\mathcal{E}_M} \|(\Delta A, \theta\Delta B)\|_F$$

$$\eta(\hat{x}_1, \hat{x}_2, \hat{x}_3) = \min_{\begin{pmatrix} \Delta B & \Delta d\\ \Delta A & \Delta b\end{pmatrix}\in\mathcal{E}_{LSE}} \left\|\begin{pmatrix} \Delta B & \Delta d\\ \Delta A & \Delta b\end{pmatrix}\right\|_F$$

$$\eta(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4) = \min_{(\Delta C, \Delta B, \Delta b)\in\mathcal{G}} \|(\Delta C, \Delta B, \Delta b)\|_F$$

where θ is a positive parameter, and the set $\mathcal{E}_M$, $\mathcal{E}_{LSE}$ and $\mathcal{G}$ are all the structure perturbation of Eq. (2),(3) and (4),respectively. Then

$$\begin{aligned}\left[\eta_F^{(\theta)}(\hat{X}, \hat{Y})\right]^2 &= \|\hat{X}\hat{X}^\dagger A\|_F^2 + \theta^2\|\hat{X}\hat{X}^\dagger(\hat{X} - B)\|_F^2 + \theta^2\|(I_m - \hat{X}\hat{X}^\dagger)(A\hat{Y} - B)\|_F^2\\ &\quad -\theta^4\|(I_m - \hat{X}\hat{X}^\dagger)(A\hat{Y} - B)\hat{Y}^T(I + \theta^2\hat{Y}\hat{Y}^T)^{-1/2}\|_F^2\end{aligned}$$

$$[\eta(\hat{x}_1, \hat{x}_2, \hat{x}_3)]^2 = \begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix}^\dagger \left[\begin{pmatrix} B\\ A\end{pmatrix}\begin{pmatrix} B\\ A\end{pmatrix}^T + \begin{pmatrix} d\\ b - \hat{x}_3\end{pmatrix}\begin{pmatrix} d\\ b - \hat{x}_3\end{pmatrix}^T\right]\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix}$$
$$+\frac{1}{1+\|\hat{x}_3\|_2^2}\left\|\left(I_{m+p} - \begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix}^\dagger\right)\begin{pmatrix} d - B\hat{x}_3\\ b - A\hat{x}_3 - \hat{x}_3\end{pmatrix}\right\|_2^2$$

$$\begin{aligned}[\eta(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4)]^2 &= \frac{1}{\|\tilde{x}_1\|_2^2}\left[\left\|C^T\tilde{x}_1\right\|_2^2 + \left\|B^T\tilde{x}_1 + \tilde{x}_2\right\|_2^2 + \left(b^T\tilde{x}_1 + \tilde{x}_2^T\tilde{x}_4\right)^2\right]\\ &+\frac{1}{1+\|\tilde{x}_3\|_2^2 + \|\tilde{x}_4\|_2^2}\left\|(I - \tilde{x}_1\tilde{x}_1^\dagger)(C\tilde{x}_3 + B\tilde{x}_4 - b)\right\|_2^2\end{aligned}$$

3. Numerical Examples

In Section 2 we have derived the expression of the backward error $\eta^{(\theta)}(\hat{x}, \hat{y})$ for the approximate solution $(\hat{x}^T, \hat{y}^T)^T$ to Eq (1). In this section, we present three numerical examples to illustrate our results. The relationship between Eq. (1) and LS shows that the solution y to Eq. (1) is the solution to LS , so the solution to Eq. (1) can be regarded as the computed solution to LS . Hence we also compare $\eta^{(\theta)}(\hat{x}, \hat{y})$ with the result of Waldén, Karlson and Sun's $\eta^{(\theta)}(\hat{y})$ which is difficult to compute. Gu [7] derives an approximation $\eta_g(\hat{y})$ to $\eta^{(0)}(\hat{y})$ that differs from it by a factor less than 2 and can be computed in $O(mn^2)$ operations, so we only need to compare $\eta^{(\theta)}(\hat{x}, \hat{y})$ with $\eta_g(\hat{y})$. All computations were performed using MATLAB, version 6.1, the relative machine precision is 2.2204×10^{-16}.

Example 3.1. Consider the structured — KKT system (1) with

$$A = \begin{pmatrix} 2 & 3\\ -1 & 1\\ 1 & 1\\ 2 & 1\end{pmatrix}, \quad b = \begin{pmatrix} -1\\ 0\\ 2\\ 3\end{pmatrix},$$

and

$$\hat{y} = \begin{pmatrix} 1.0000 \\ -0.5000 \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} -1.5000 \\ 1.5000 \\ 1.5000 \\ 1.5000 \end{pmatrix}$$

is a computed solution to the structured — KKT system (1). Some numerical results for the backward error are listed in Table 1.

Table 1

θ	$\eta_g(\hat{y})$	$\eta^{(\theta)}(\hat{x}, \hat{y})$	$\tau(\hat{z})$
0.01	1.4134e-15	4.4434e-16	8.4339e-17
0.1	1.4134e-15	4.6781e-16	8.4339e-17
1	1.4134e-15	1.1176e-15	8.4339e-17
10	1.4134e-15	6.2989e-15	8.4339e-17

Example 3.2. Consider the structured — KKT system (1) with

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1+\varepsilon \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 0 \\ 1 - 10^3\varepsilon \end{pmatrix}, \quad \varepsilon = \begin{cases} 10^{-4} \\ 1 \end{cases}$$

and

$$\hat{y} = \begin{pmatrix} 1.0010 \\ -1.0000 \end{pmatrix} \times 10^3, \quad \hat{x} = \begin{pmatrix} 1.0000 \\ -1.0000 \\ -0.0000 \end{pmatrix}$$

is a computed solution to the structured — KKT system (1). Some numerical results for the backward error are listed in Table 2 and Table 3.

Table 2 $\varepsilon = 10^{-4}$

θ	$\eta_g(\hat{y})$	$\eta^{(\theta)}(\hat{x}, \hat{y})$	$\tau(\hat{z})$
0.01	3.7000e-14	5.2408e-13	4.3453e-16
0.1	3.7000e-14	5.2245e-12	4.3453e-16
1	3.7000e-14	5.2243e-11	4.3453e-16
10	3.7000e-14	5.2243e-10	4.3453e-16

Table 3 $\varepsilon = 1$

θ	$\eta_g(\hat{y})$	$\eta^{(\theta)}(\hat{x}, \hat{y})$	$\tau(\hat{z})$
0.01	3.9359e-16	6.6145e-13	1.3433e-16
0.1	3.9359e-16	4.0247e-12	1.3433e-16
1	3.9359e-16	3.9903e-11	1.3433e-16
10	3.9359e-16	3.9900e-10	1.3433e-16

Example 3.3. Consider the structured — KKT system (1) with

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 1 & 2 & 3 & -1 \\ 2 & 3 & -1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 1 \\ 4 \\ -6 \end{pmatrix},$$

and

$$\hat{y} = \begin{pmatrix} -0.7882 \\ -0.8902 \\ 1.9137 \\ -0.1608 \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} -0.6667 \\ -0.6667 \\ 0.6667 \\ 0.0000 \end{pmatrix}$$

is a computed solution to the structured — KKT system (1). Some numerical results for the backward error are listed in Table 4.

Table 4

θ	$\eta_g(\hat{y})$	$\eta^{(\theta)}(\hat{x}, \hat{y})$	$\tau(\hat{z})$
0.01	1.0084e-15	3.4402e-15	1.5643e-16
0.1	1.0084e-15	3.4702e-15	1.5643e-16
1	1.0084e-15	3.4702e-15	1.5643e-16
10	1.0084e-15	3.5002e-14	1.5643e-16

From the results listed in Table 1-Table 4, we get the following conclusions:
1. the solution to the structured —- KKT system (1) obtained by using the stable algorithm can be regarded as the computed solution to the linear least squares problem (5).
2. The stable solution to the structured —- KKT system (1) is not necessarily strongly stable.

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