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A Spectral Time-Domain Method for Computational Electrodynamics

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Abstract. Ever since its introduction by Kane Yee over forty years ago, the finite-difference time-domain (FDTD) method has been a widely-used technique for solving the time-dependent Maxwell's equations that has also inspired many other methods. This paper presents an alternative approach to these equations in the case of spatially-varying electric permittivity and/or magnetic permeability, based on Krylov subspace spectral (KSS) methods. These methods have previously been applied to the variable-coefficient heat equation and wave equation, and have demonstrated high-order accuracy, as well as stability characteristic of implicit time-stepping schemes, even though KSS methods are explicit. KSS methods for scalar equations compute each Fourier coefficient of the solution using techniques developed by Golub and Meurant for approximating elements of functions of matrices by Gaussian quadrature in the spectral, rather than physical, domain. We show how they can be generalized to coupled systems of equations, such as Maxwell's equations, by choosing appropriate basis functions that, while induced by this coupling, still allow efficient and robust computation of the Fourier coefficients of each spatial component of the electric and magnetic fields. We also discuss the application of block KSS methods to problems involving non-self-adjoint spatial differential operators, which requires a generalization of the block Lanczos algorithm of Golub and Underwood to unsymmetric matrices.

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1 Introduction

We consider Maxwell's equation on the rectangle $[0, 2\pi]^3$, with periodic boundary con-

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ditions. Assuming nonconductive material with no losses, we have

$$\operatorname{div} \hat{\mathbf{E}} = 0, \qquad \operatorname{div} \hat{\mathbf{H}} = 0, \tag{1.1}$$

$$\operatorname{curl} \hat{\mathbf{E}} = -\mu \frac{\partial \hat{\mathbf{H}}}{\partial t}, \qquad \operatorname{curl} \hat{\mathbf{H}} = \varepsilon \frac{\partial \hat{\mathbf{E}}}{\partial t}, \tag{1.2}$$

where $\hat{\mathbf{E}}$, $\hat{\mathbf{H}}$ are the vectors of the electric and magnetic fields, and ε, μ are the electric permittivity and magnetic permeability, respectively. We assume that these two functions are smoothly varying in space.

By taking the curl of both sides of (1.2), we decouple the vector fields $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ and obtain the equations

$$\mu\varepsilon \frac{\partial^2 \hat{\mathbf{E}}}{\partial t^2} = \Delta \hat{\mathbf{E}} + \mu^{-1} \operatorname{curl} \hat{\mathbf{E}} \times \nabla \mu, \tag{1.3}$$

$$\mu\varepsilon \frac{\partial^2 \hat{\mathbf{H}}}{\partial t^2} = \Delta \hat{\mathbf{H}} + \varepsilon^{-1} \operatorname{curl} \hat{\mathbf{H}} \times \nabla \varepsilon. \tag{1.4}$$

In paper [26], Yee proposed the original finite-difference time-domain method for solving Eqs. (1.1) and (1.2). This method uses a staggered grid to avoid solving simultaneous equations for $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$, and also removes numerical dissipation. However, because it is an explicit finite-difference scheme, its time step is constrained by the CFL condition. Nonetheless, it remains a widely used method to this day, and has inspired a host of related methods, including, for example, several that are based on spatial discretizations other than finite differences, such as a pseudospectral time-domain (PSTD) method [20], an FDTD-FEM hybrid method [22], and a one-step algorithm based on Chebyshev polynomial approximations [5]. In this paper, we introduce a new time-domain method for these equations.

In [18], a class of methods, called Krylov subspace spectral (KSS) methods, was introduced for the purpose of solving parabolic variable-coefficient PDE. These methods are based on techniques developed by Golub and Meurant in [7] for approximating elements of a function of a matrix by Gaussian quadrature in the *spectral* domain. In [11, 14], these methods were generalized to the second-order wave equation, for which these methods have exhibited even higher-order accuracy.

It has been shown in these references that KSS methods, by employing different approximations of the solution operator for each Fourier coefficient of the solution, achieve higher-order accuracy in time than other Krylov subspace methods (see, e.g., [13]) for stiff systems of ODE, and, as shown in [14], they are also quite stable, considering that they are explicit methods. In [15, 16], the accuracy and robustness of KSS methods were enhanced using block Gaussian quadrature.

It is our hope that the high-order accuracy achieved for the scalar wave equation can be extended to systems of coupled wave equations such as those described by Maxwell's equations. Section 2 reviews the main properties of KSS methods, including block KSS methods, as applied to the parabolic problems for which they were originally designed. Section 3 reviews their application to the wave equation, including previous convergence analysis. In Section 4, we discuss the modifications that