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Viscoelastic Models for Passive Arterial Wall Dynamics

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Abstract. This paper compares two models predicting elastic and viscoelastic properties of large arteries. Models compared include a Kelvin (standard linear) model and an extended 2-term exponential linear viscoelastic model. Models were validated against in-vitro data from the ovine thoracic descending aorta and the carotid artery. Measurements of blood pressure data were used as an input to predict vessel cross-sectional area. Material properties were predicted by estimating a set of model parameters that minimize the difference between computed and measured values of the cross-sectional area. The model comparison was carried out using generalized analysis of variance type statistical tests. For the thoracic descending aorta, results suggest that the extended 2-term exponential model does not improve the ability to predict the observed cross-sectional area data, while for the carotid artery the extended model does statistically provide an improved fit to the data. This is in agreement with the fact that the aorta displays more complex nonlinear viscoelastic dynamics, while the stiffer carotid artery mainly displays simpler linear viscoelastic dynamics.

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Key words: Arterial wall modeling, dynamic viscoelastic models, inverse problems, statistical analysis for model comparison.

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1 Introduction

The arterial walls of the cardiovascular system are known to display complex mechanical responses under physiological conditions. The typical arterial wall consists of three layers: an innermost layer, "the intima", mainly composed of endothelial cells, a middle layer, "the media", composed of elongated smooth muscle cells, elastin and collagen, and an outer layer "the adventitia" comprising a varied number of elastic sheets, bundles of collagen fibrils, and a network of elastic fibrils. These layers and the composition of the layers are shown in Fig. 1.

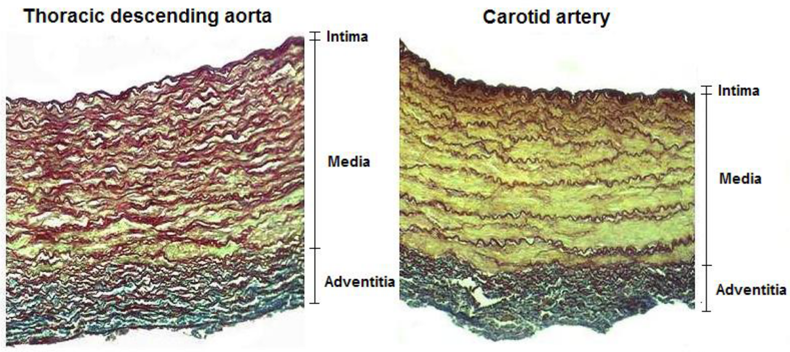

Figure 1: Histological slices displaying a cross section of the arterial wall for the thoracic descending aorta (left) and the carotid artery (right) from ovine arteries. The vessels were stained with orcein using the Cajal-Gallengo method, which allows discrimination of the three main wall components that determine the arterial biomechanical behavior: smooth muscle cells (yellow), elastin (dark red), and collagen (blue). Note that the carotid artery has a higher proportion of smooth muscle cells ($\approx 60\%$) than the thoracic descending aorta ($\approx 40\%$), while the aorta has more elastin fibers than the carotid artery.

In this study, our focus is on viscoelastic modeling of the passive dynamic mechanical responses of the arterial wall under in-vitro conditions that mimic in-vivo physiological conditions. Within this context, two models will be evaluated by comparison to experimental pressure-area data for two arteries: the thoracic descending aorta and the carotid artery. The aorta is the largest artery in the cardiovascular system while the carotid artery is significantly smaller. For sheep arteries, the thoracic descending aorta typically has a diameter of 2 cm, while the carotid artery has a diameter of approximately 0.8 cm. The collagen-elastin-smooth muscle cell composition of these two types of arteries is different; the aorta contains a significant amount of elastin fibers and fewer smooth muscle cells (approximately 40% of the aortic vessel wall is composed of smooth muscle cells), while the carotid artery has less elastin and significantly more smooth muscle cells (approximately 60%) [15]. Another difference between the two vessels is that the separation between elastin fibers, collagen fibers, and smooth muscle cells are more distinguished for the carotid artery than for the aorta; see Fig. 1.

For both vessels, the media layer gives rise to the majority of the vessel's viscoelastic behavior, and it is the presence and organization of fibers and smooth muscle cells