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Finite Element Analysis of Maxwell's Equations in Dispersive Lossy Bi-Isotropic Media

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Dedicated to Graeme Fairweather on the occasion of his 70^{th} birthday.

Abstract. In this paper, the time-dependent Maxwell's equations used to modeling wave propagation in dispersive lossy bi-isotropic media are investigated. Existence and uniqueness of the modeling equations are proved. Two fully discrete finite element schemes are proposed, and their practical implementation and stability are discussed.

AMS subject classifications: 65N30, 35L15, 78M10

Key words: Maxwell's equations, bi-isotropic media, finite element method.

1 Introduction

The research on numerical analysis and modeling of electromagnetic wave propagation in dispersive media (especially metamaterials) has been a subject of increasing interest over the recent years (cf. [1,6,10–14,16,19–21] and references cited therein). In this paper, we consider the wave propagation problem in dispersive lossy bi-isotropic (BI) media, which are characterized by more complicated constitutive relations than those classical dispersive media models such as Debye and Lorentz models [10]. In BI media, the magnetic and electric fields are coupled. Electromagnetic waves in such media have some interesting characteristics such as optical rotatory dispersion [15].

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Though some FDTD schemes (cf. [7]) have been developed for solving BI media, to our best knowledge, there is no rigorous mathematical analysis (such as the existence and uniqueness) of this model. Furthermore, to overcome the disadvantage of FDTD schemes for complex geometric problems, it is interesting to develop some finite element method for modeling wave propagation in BI media. Our major goal of this paper is to initiate the analysis of these new modeling equations and develop some efficient finite element methods to solve them.

In this paper, we denote C (sometimes with a sub-index) a generic constant independent of the mesh size h and the time step size Δt. We also use some common notations [17]:

$$\begin{aligned} &H(div;\Omega)=\{\boldsymbol{v}\in(L^2(\Omega))^3:\ \nabla\cdot\boldsymbol{v}\in(L^2(\Omega))^3\},\\ &H(curl;\Omega)=\{\boldsymbol{v}\in(L^2(\Omega))^3:\ \nabla\times\boldsymbol{v}\in(L^2(\Omega))^3\},\\ &H_0(curl;\Omega)=\{\boldsymbol{v}\in H(curl;\Omega):\ \boldsymbol{n}\times\boldsymbol{v}=\boldsymbol{0}\ \text{ on }\ \partial\Omega\}, \end{aligned}$$

for any bounded Lipschitz polyhedral domain Ω in $\mathcal{R}^3$ with connected boundary $\partial\Omega$. Moreover, we let $(H^\alpha(\Omega))^3$ be the standard Sobolev space equipped with norm $\|\cdot\|_\alpha$. When $\alpha=0$, we just denote $\|\cdot\|_0$ for the $(L^2(\Omega))^3$ norm.

The rest of the paper is organized as follows. In Section 2, we first present the time-dependent governing equations for modeling wave propagation in BI media. Then we prove the existence and uniqueness of the modeling equations. We also present a stability result. In Section 3, we develop two fully-discrete finite element schemes for solving the BI media model equations. Solvability, stability of these schemes are discussed. Finally, we conclude the paper in Section 4.

2 The governing equations

The description of the dispersive lossy BI media is given by the constitutive relations [15]:

$$\boldsymbol{D}=\epsilon(\omega)\boldsymbol{E}+\sqrt{\epsilon_0\mu_0}(\chi-i\kappa(\omega))\boldsymbol{H}, \tag{2.1a}$$

$$\boldsymbol{B}=\mu(\omega)\boldsymbol{H}+\sqrt{\epsilon_0\mu_0}(\chi+i\kappa(\omega))\boldsymbol{E}, \tag{2.1b}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ denote the electric field and magnetic field, $\boldsymbol{D}$ and $\boldsymbol{B}$ denote the electric and magnetic flux densities respectively, ϵ_0 and μ_0 are the vacuum permittivity and permeability respectively, the number $i=\sqrt{-1}$, $\chi\geq 0$ is the nonreciprocity parameter, and $\kappa(\omega)$ is the chirality parameter. Furthermore, the permittivity $\epsilon(\omega)$ and permeability $\mu(\omega)$ depend on the wave frequency ω. Experiments found that a Condon model can be used to describe the frequency of the chirality $\kappa(\omega)$, and both $\epsilon(\omega)$ and $\mu(\omega)$ follow a second-order Lorentz model. Since the resonance frequencies of $\kappa(\omega)$, $\epsilon(\omega)$ and $\mu(\omega)$ are found to be very close in experiments, in practice they are assumed to be the same, in which case, the frequency domain constitutive relations (2.1a)-(2.1b) are expressed as