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Some Approximation Properties of Certain q-Baskakov-Beta Operators

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Abstract. In this paper, we propose the q analogue of modified Baskakov-Beta operators. The Voronovskaja type theorem and some direct results for the above operators are discussed. The rate of convergence and weighted approximation by the operators are studied.

Key Words: q-Baskakov-Beta operators, rate of convergence, weighted approximation.

AMS Subject Classifications: 41A25, 41A35

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1 Introduction

In recent years, the application of q calculus is the most interesting areas of research in the approximation theory (e.g., [1]). Lupaş [2] and Phillips [3] proposed generalizations of Bernstein polynomials based on the q-integers. More results on q-Bernstein operators were investigated (e.g., [4, 5]). Gupta and Aral [6, 7] proposed certain q-analogues of the Baskakov operators and studied some approximation properties of q-Baskakov operators.

In approximation theory the Durrmeyer type integral modification of certain discrete operators is also an active area of research. Cai [8] investigated the convergence of modification of Durrmeyer type q-Baskakov operators. Gupta and collaborators (see [9–13], etc.) introduced several important q analogues of different Durrmeyer type operators and established interesting approximation results. In [14], Gupta observed that the Baskakov operators by taking weight functions of Beta basis function give better approximatin results. Wang [15] also estimated asymptotic formula for Baskakov Beta operators in generalized form.

The aim of this paper is to study the approximation properties of cerntain generalization of Baskakov Beta operators, based on q-integer. We first recall some concept of

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q-calculus, which can be found in [16]. In what follows, q is a real number satisfying $0<q<1$.

For $k\in\mathbb{N}$, the q integer and q factorial are given by

$$[k]_q=\frac{1-q^k}{1-q},\qquad [k]_q!=\begin{cases}[k]_q[k-1]_q\cdots[1]_q, & k\geq 1,\\ 1, & k=0.\end{cases}$$

The q-binomial coefficients are defined as

$$\begin{bmatrix} n\\ k\end{bmatrix}=\frac{[n]_q!}{[k]_q![n-k]_q!},\qquad 0\leq k\leq n. \tag{1.1}$$

The q-Pochhammer symbol is defined as

$$(-x,q)_n=(1+x)(1+qx)\cdots(1+q^{n-1}x)=\prod_{j=0}^{n-1}(1+q^jx).$$

The q-Jackson integrals and the q-improper integrals are defined as (see [17, 18])

$$\int_0^a f(x)d_qx=(1-q)a\sum_{n=0}^{\infty}f(aq^n)q^n,\qquad a>0,$$

and

$$\int_0^{\infty/A} f(x)d_qx=(1-q)\sum_{n=-\infty}^{\infty}f\Big(\frac{q^n}{A}\Big)\frac{q^n}{A},\qquad A>0, \tag{1.2}$$

provided the sum converge absolutely.

The q-Gamma integral (see [19]) is defined by

$$\Gamma_q(t)=\int_0^{\frac{1}{1-q}}x^{t-1}E_q(-qx)d_qx,\quad t>0, \tag{1.3}$$

where

$$E_q(x)=\sum_{n=0}^{\infty}q^{n(n-1)/2}\frac{x^n}{[n]_q!}.$$

Also $\Gamma_q(t+1)=[t]_q\Gamma_q(t)$, $\Gamma_q(1)=1$.

The q-Beta integral (see [19]) is defined by

$$B_q(t,s)=K(A,t)\int_0^{\infty/A}\frac{x^{t-1}}{(1+x)_q^{t+s}}d_qx, \tag{1.4}$$

where

$$K(x,t)=\frac{1}{x+1}x^t\Big(1+\frac{1}{x}\Big)_q^t(1+x)_q^{1-t}.$$