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Toeplitz Type Operator Associated to Singular Integral Operator with Variable Kernel on Weighted Morrey Spaces

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Abstract. Suppose $T^{k,1}$ and $T^{k,2}$ are singular integrals with variable kernels and mixed homogeneity or $\pm I$ (the identity operator). Denote the Toeplitz type operator by

$$T^b = \sum_{k=1}^{Q} T^{k,1} M^b T^{k,2},$$

where $M^b f = bf$. In this paper, the boundedness of T^b on weighted Morrey space are obtained when b belongs to the weighted Lipschitz function space and weighted BMO function space, respectively.

Key Words: Toeplitz type operator, singular integral operator, variable Calderón-Zygmund kernel, weighted BMO function, weighted Lipschitz function, weighted Morrey space.

AMS Subject Classifications: 42B20, 40B35

1 Introduction

The classical Morrey spaces, introduced by Morrey [1] in 1938, have been studied intensively by various authors, and it, together with weighted Lebesgue spaces play an important role in the theory of partial differential equations, see [2, 3]. The boundedness of the Hardy-Littlewood maximal operator, singular integral operator, fractional integral operator and commutator of these operators in Morrey spaces have been studied by Chiarenza and Frasca in [4]. Komori and Shirai [5] introduced a version of the weighted Morrey space $L^{p,\kappa}(\omega)$, which is a natural generalization of the weighted Lebesgue space $L^p(\omega)$.

As the development of singular integral operators, their commutators have been well studied [6–8]. In [7], the authors proved that the commutators $[b,T]$, which generated by

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Calderón-Zygmund singular integral operator and BMO functions, are bounded on L^p for $1<p<\infty$. The commutator generated by the Calderón-Zygmund operator T and a locally integrable function b can be regarded as a special case of the Toeplitz operator

$$T^b=\sum_{k=1}^{Q} T^{k,1}M^bT^{k,2}, \tag{1.1}$$

where $T^{k,1}$ and $T^{k,2}$ are the Calderón-Zygmund operators or $\pm I$ (the identity operator), $M^bf=bf$. When $b\in BMO$, the L^p-boundedness of T^b was discussed, see [9,10]. In [11,12], the authors studied the boundedness of T^b on Morrey spaces.

Let $K(x,\xi):\ \mathbb{R}^n\times\mathbb{R}^n\setminus\{0\}\to\mathbb{R}$ be a variable kernel with mixed homogeneity. The singular integral operator is defined by

$$Tf(x)=p.v.\int_{\mathbb{R}^n}K(x,x-y)f(y)dy. \tag{1.2}$$

The variable kernel $K(x,\xi)$ depends on some parameter x and possesses good properties with respect to the second variable ξ, which was firstly introduced by Fabes and Rieviéve in [13]. They generalized the classical Calderón-Zygmund kernel and the parabolic kernel studied by Jones in [14]. By introducing a new metric ρ, Fabes and Rieviéve studied (1.2) in $L^p(\mathbb{R}^n)$, where $\mathbb{R}^n$ was endowed with the topology induced by ρ and defined by ellipsoids.

By using this metric ρ, Softova in [15] obtained that the integral operator (1.2) and its commutator were continuous in generalized Morrey space $L^{p,\omega}(\mathbb{R}^n)$, $1<p<\infty$, ω satisfying suitable conditions. Ye and Zhu in [16] discussed the continuity of (1.2) and its multilinear commutator in the weighted Morrey spaces $L^{p,\kappa}(\omega)$, $1<p<\infty$, $0<\kappa<1$, and ω is A_p weight.

Suppose $T^{k,1}$ and $T^{k,2}$ are singular integrals whose kernels are variable kernel with mixed homogeneity or $\pm I$ (the identity operator). In this paper, we study the boundedness of Toeplitz operators T^b as (1.1) in weighted Morrey spaces when b belongs to weighted Lipschitz spaces and weighted BMO spaces, respectively. The main results are as follows.

Theorem 1.1. *Suppose that T^b is a Toeplitz type operator associated to singular integral operator with variable kernel, $\omega\in A_1$, and $b\in Lip_{\beta,\omega}$. Let $0<\kappa<p/q$, $1<p<n/\beta$ and $1/q=1/p-\beta/n$. If $T^1(f)=0$ for any $f\in L^{p,\kappa}(\omega)$, then there exists a constant $C>0$ such that,*

$$\|T^b(f)\|_{L^{q,\kappa q/p}(\omega^{1-q},\omega)}\le C\|b\|_{Lip_{\beta,\omega}}\|f\|_{L^{p,\kappa}(\omega)}.$$

Theorem 1.2. *Suppose that T^b is a Toeplitz type operator associated to singular integral operator with variable kernel, $\omega\in A_1$, and $b\in BMO(\omega)$. Let $1<p<\infty$, and $0<\kappa<1$. If $T^1(f)=0$ for any $f\in L^{p,\kappa}(\omega)$, then there exists a constant $C>0$ such that,*

$$\|T^b(f)\|_{L^{p,\kappa}(\omega^{1-p},\omega)}\le C\|b\|_{*,\omega}\|b\|_{L^{p,\kappa}(\omega)}.$$