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SHARP MAXIMAL FUNCTION ESTIMATE AND WEIGHTED INEQUALITIES FOR MAXIMAL MULTILINEAR SINGULAR INTEGRALS

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Abstract. For maximal multilinear Calderón-Zygmund singular integral operators, the sharp maximal function estimate and some weighted norm inequalities are obtained.

Key words: *multilinear Calderón-Zygmund operator, sharp maximal function, weighted norm inequality*

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1 Introduction

Let T be a multilinear operator initially defined on the m-fold product of Schwartz spaces and taking values into the space of tempered distributions,

$$T:\mathcal{S}(\mathbf{R}^n)\times\cdots\times\mathcal{S}(\mathbf{R}^n)\longrightarrow\mathcal{S}'(\mathbf{R}^n).$$

We say that T is an m-linear Calderón-Zygmund operator, if for some $1\le q_j<\infty$, it extends to a bounded multilinear operator from $L^{q_1}\times\cdots\times L^{q_m}$ to L^q, where $1/q=1/q_1+\cdots+1/q_m$, and if there exists a function K, defined off the diagonal $x=y_1=\cdots=y_m$ in $(\mathbf{R}^n)^{m+1}$, for $\vec{f}=(f_1,\cdots,f_m)$, satisfying

$$T(\vec{f})(x)=\int_{(\mathbf{R}^n)^m}K(x,y_1,\cdots,y_m)f_1(y_1)\cdots f_m(y_m)\mathrm{d}\vec{y}$$

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for all $x \notin \bigcap_{j=1}^{m} \mathrm{supp} f_j$, where $d\vec{y} = dy_1 \cdots dy_m$ and $\vec{y} = (y_1, \cdots, y_m)$;

$$|K(y_0, y_1, \cdots, y_m)| \le \frac{A}{\Big(\sum\limits_{k,j=0}^{m} |y_k - y_l|\Big)^{mn}}; \tag{1}$$

and

$$|K(y_0, \cdots, y_j, \cdots, y_m) - K(y_0, \cdots, y_j', \cdots, y_m)| \le \frac{A|y_j - y_j'|^{\gamma}}{\Big(\sum\limits_{k,j=0}^{m} |y_k - y_l|\Big)^{mn+\gamma}}, \tag{2}$$

for some $\gamma > 0$ and all $0 \le j \le m$, whenever $|y_j - y_j'| \le \frac{1}{2} \max_{0 \le k \le m} |y_j - y_k|$.

The multilinear Calderón-Zygmund theory has been developed by Grafakos and Torres[1][2]. These articles and the references therein contain the background and applications about this subject. It was shown in [1] that if $1/r = 1/r_1 + \cdots + 1/r_m$, then an m-linear Calderón-Zygmund operator satisfies

$$T : L^{r_1} \times \cdots \times L^{r_m} \longrightarrow L^{r} \tag{3}$$

when $1 < r_j < \infty$ for all $j = 1, \cdots, m$; and

$$T : L^{r_1} \times \cdots \times L^{r_m} \longrightarrow L^{r,\infty}, \tag{4}$$

when $1 \le r_j < \infty$ for all $j = 1, \cdots, m$, and at least one $r_j = 1$. In particular

$$T : L^{1} \times \cdots \times L^{1} \longrightarrow L^{1/m,\infty}. \tag{5}$$

Given $\varepsilon > 0$, for $x \in \mathbf{R}^n$, define the truncated operator by

$$T_{\varepsilon}(\vec{f})(x) = \int_{|x-y_1|^2+\cdots+|x-y_m|^2>\varepsilon^2} K(x,\vec{y}) f_1(y_1) \cdots f_m(y_m) \mathrm{d}\vec{y}$$

and the associated maximal operator by

$$T^{*}(\vec{f})(x) = \sup_{\varepsilon>0} |T_{\varepsilon}(\vec{f})(x)|.$$

Grafakos and Torres in [2] proved that the maximal operator T^* satisfies the same boundedness as T in (3), (4), (5) and some weighted norm inequalities.

Recently, Lerner, Ombrosi, Pérez and Trujillo-González[3] defined a new multilinear maximal function associated to the m-linear Calderón-Zygmund operator as

$$\mathcal{M}(\vec{f})(x) = \sup_{Q \ni x} \prod_{j=1}^{m} \frac{1}{|Q|} \int_{Q} |f_j(y_j)| \mathrm{d}y_j,$$

and developed a $A_{\vec{P}}$ weighted theory for the this multilinear maximal function and multilinear Calderón-Zygmund operators.

Motivated by the work in [4], we consider here the sharp maximal function estimate and weighted norm inequalities for the maximal operator T^*.