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ON APPROXIMATION AND GENERALIZED TYPE OF ANALYTIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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Abstract. In the present paper, we study the polynomial approximation of analytic functions of several complex variables. The characterizations of generalized type of analytic functions of several complex variables have been obtained in terms of approximation and interpolation errors.

Key words: *analytic function, Siciak extremal function, generalized type, approximation errors, imterpolation errors*

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1 Introduction

The concept of generalized order for analytic functions was given by Seremeta[3] and Janik[2]. Hence, let L^0 denote the class of functions h satisfying the following conditions:

(i) $h(x)$ is defined on $[a,\infty)$ and is positive, strictly increasing, differentiable and tends to ∞ as $x \to \infty$;

(ii) $\lim\limits_{x\to\infty} \dfrac{h\{(1+1/\psi(x))x\}}{h(x)} = 1;$
for every function $\psi(x)$ such that $\psi(x) \to \infty$ as $x \to \infty$.

Let Λ denote the class of functions h satisfying the condition (i) and

(iii) $\lim\limits_{x\to\infty} \dfrac{h(cx)}{h(x)} = 1,$
for every $c > 0$, that is $h(x)$ is slowly increasing.

Let K be a compact set in C^N such that the Siciak extremal function of K

$$\Phi_{\mathrm{K}}(\mathrm{z}) = \sup[|(p(z)|^{1/n} : p\text{−polynomial}, \deg p \le n, ||p||_K \le 1, n \ge 1], z \in C^N,$$

is continuous, $||.||_K$ being the sup norm on K (see [1] and [2]).

Let $g : C^N \to C, N \ge 1$, be a function analytic in $K_R = \{z \in C^N : \Phi_K(z) < R\}, R > 1$. Now for $1 < r < R$, we put $S(r,g) = \sup\{|g(z)| : \Phi_K(z) = r\}$. For $\alpha \in \Lambda$ and $\beta \in L^0$ Seremeta [4] introduced the concept of generalized order of entire functions. For $\alpha, \beta \in \Lambda$, Janik [2] defined the generalized order of analytic function $g(z)$ as

$$\rho(\alpha,\beta,g) = \lim_{r\to R}\sup \frac{\alpha\,[\log^+ S(r,g)]}{\beta\,[R/(R-r)]}.$$

He also obtained the characterization of $\rho(\alpha,\beta,g)$ in terms of approximation and interpolation errors.

In this note we define the generalized type of analytic function $g(z)$ and obtain the characterization of $\sigma(\alpha,\beta,\rho,g)$ in terms of approximation and interpolation errors. Thus let the functions α,β and $\gamma \in \Lambda$. Then for $0 < \rho < \infty$, we define the generalized type of analytic function $g(z)$ as

$$\sigma(\alpha,\beta,\rho,g) = \lim_{r\to R}\sup \frac{\alpha\,[\log^+ S(r,g)]}{\beta\,\{[\gamma\{R/(R-r)\}]^{\rho}\}}.$$

Given a function f defined and bounded on K, we put for $n = 1, 2, \cdots$

$$E_n^1(f,K) = ||f - t_n||_K;$$
$$E_n^2(f,K) = ||f - l_n||_K;$$
$$E_{n+1}^3(f,K) = ||l_{n+1} - l_n||_K;$$

where t_n denotes the n^{th} Chebyshev polynomial of the best approximation to f on K and l_n denotes the n^{th} Lagrange interpolation polynomial for f with nodes at extremal points of K (see [1] and [2]). Before proving the main result we state and prove a lemma.

Lemma 1.1. *Let $\alpha(x)$, $\beta^{-1}(x)$, $\gamma(x) \in \Lambda$ and K be a compact set in C^N such that Φ_K is locally bounded in C^N. Set $F(x,\mu,\rho) = \gamma^{-1}\{[\beta^{-1}(\mu\,\alpha(x))]^{1/\rho}\}$. Assume that for all positive numbers μ and ρ*

$$\lim_{x\to\infty} \frac{d[\log(F(x,\mu,\rho))}{d(\log x)} < 1.$$

Let $(p_n)_{n\in N}$ be a sequence of polynomials in C^N such that

(i) $\deg p_n \le n$, $n \in \mathbf{N}$;

(ii) *there exists $n_0 \in N$ and $R > 1$, such that for all $n \ge n_0$*

$$\log^+(||p_n||R^n) \le n\,\frac{\rho+1}{\rho}\left[\gamma^{-1}\left\{\left[\beta^{-1}\left\{\mu^{-1}\alpha(n/\rho)\right\}\right]^{1/(\rho+1)}\right\}\right]^{-1}.$$