Anal. Theory Appl.
Vol. 28, No. 4 (2012), 363–376
DOI : 10.3969/j.issn.1672-4070.2012.04.005

A CLASS OF HARMONIC STARLIKE FUNCTIONS WITH RESPECT TO SYMMETRIC POINTS ASSOCIATED WITH WRIGHT GENERALIZED HYPERGEOMETRIC FUNCTION

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Received Apr. 27, 2012

Abstract. Making use of Wright operator we introduce a new class of complex-valued harmonic functions with respect to symmetric points which are orientation preserving, univalent and starlike. We obtain coefficient conditions, extreme points, distortion bounds, and convex combination.

Key words: *harmonic, univalent, Wright operator, symmetric point*

AMS (2010) subject classification: 30C45

1 Introduction

Denote by $\mathcal{H}$ the family of functions

$$f = h + \overline{g}\,, \tag{1.1}$$

which are analytic univalent and sense-preserving in the unit disc $U = \{z : |z| < 1\}$. So that f is normalized by $f(0) = f_z(0) - 1 = 0$. Thus, for $f = h + \overline{g} \in \mathcal{H}$, we may express the analytic functions h and g in the forms

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k, \qquad g(z) = \sum_{k=1}^{\infty} b_k z^k \qquad |b_1| < 1\,. \tag{1.2}$$

where h and g are analytic in D. We call h the analytic part and g the co-analytic part of f. A necessary and sufficient condition for f to be locally univalent and sense-preserving in $\mathcal{H}$ is that $|h'(z)| > |g'(z)|$ in $\mathcal{H}$ (see [4]).Hence

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k + \sum_{k=1}^{\infty} \overline{b_k z^k},\ |b_1| < 1. \tag{1.3}$$

We denote $\overline{\mathcal{H}}$ the subclass of $\mathcal{H}$ consists of harmonic functions $f = h + \overline{g}$ of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k + \sum_{k=1}^{\infty} \overline{b_k z^k},\ |b_1| < 1. \tag{1.4}$$

Let the Hadamard product (or convolution) of two power series $\Phi(z) = z + \sum_{k=2}^{\infty} \phi_k z^k$ and $\Psi(z) = z + \sum_{k=2}^{\infty} \psi_k z^k$ be defined by

$$(\Phi * \Psi)(z) = z + \sum_{k=2}^{\infty} \phi_k \psi_k z^k = (\Psi * \Phi)(z).$$

Let $\alpha_1, A_1, \cdots, \alpha_q, A_q$ and $\beta_1, B_1, \cdots, \beta_s, B_s$ $(q, s \in \mathbf{N})$ be positive and real parameters such that

$$1 + \sum_{j=1}^{s} B_j - \sum_{j=1}^{q} A_j \geq 0.$$

The Wright generalized hypergeometric function[19] (see also [12)

$${}_q\Psi_s\left[(\alpha_1, A_1), ..., (\alpha_q, A_q); (\beta_1, B_1), ..., (\beta_s, B_s); z\right] = {}_q\Psi_s\left[(\alpha_i, A_i)_q; (\beta_i, B_i)_s; z\right]$$

is defined by

$${}_q\Psi_s\left[(\alpha_i, A_i)_q; (\beta_i, B_i)_s; z\right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{q} \Gamma(\alpha_i + nA_i)}{\prod_{i=1}^{s} \Gamma(\beta_i + nB_i)} \frac{z^n}{n!}, \qquad z \in U.$$

If $A_i = 1 (i = 1, \cdots, q)$ and $B_i = 1 (i = 1, \cdots, s)$, we have the relationship:

$$\Omega_q\Psi_s\left[(\alpha_i, 1)_q; (\beta_i, 1)_s; z\right] = {}_qF_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z),$$

where ${}_qF_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z)$ is the generalized hypergeometric function (see for details [6], [7], [8], [9], [13]) and

$$\Omega = \frac{\prod_{i=1}^{s} \Gamma(\beta_i)}{\prod_{i=1}^{q} \Gamma(\alpha_i)}. \tag{1.5}$$