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THE BOUNDEDNESS FOR A CLASS OF ROUGH FRACTIONAL INTEGRAL OPERATORS ON VARIABLE EXPONENT LEBESGUE SPACES

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Abstract. In this paper, we will discuss the behavior of a class of rough fractional integral operators on variable exponent Lebesgue spaces,and establish their boundedness from $L^{p_1(\cdot)}(\mathbf{R}^n)$ to $L^{p_2(\cdot)}(\mathbf{R}^n)$.

Key words: *fractional integral, rough kernel, variable exponent Lebesgue space*

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1 Introduction and Main Results

In 1931, the variable exponent Lebesgue space has been first proposed in [1] by Polish mathematician Orlicz. In the last years the space has attracted more and more attention, see for example [1-5]. The main motivation for studying the space is applications to models of elasticity theory, fluid mechanics and differential equation with non-standard growth, see for example [6-8].

Let S^{n-1} denote the unit sphere in the Euclidean n-dimensional space $\mathbf{R}^n$. Suppose that $\Omega \in L^s\left(S^{n-1}\right)$, $s > \dfrac{n}{n-\alpha}$, is homogeneous of degree zero on $\mathbf{R}^n$. Then the fractional integral operator $T_{\Omega,\alpha}$ with a rough kernel is defined by

$$T_{\Omega,\alpha} f(x) = \int_{\mathbf{R}^n} \frac{\Omega(x-y)}{|\, x-y\, |^{n-\alpha}} f(y)\mathrm{d}y,$$

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and in the case $\Omega\equiv 1$, $T_{\Omega,\alpha}$ is the fractional integral operator (or Riesz potential operator)

$$I_\alpha f(x)=\int_{\mathbf{R}^n}\frac{|f(y)|}{|x-y|^{n-\alpha}}\mathrm{d}y.$$

The corresponding fractional maximal operator with a rough kernel is defined by

$$M_{\Omega,\alpha}f(x)=\sup_{Q\ni x}\frac{1}{|Q|^{1-\frac{\alpha}{n}}}\int_Q|\Omega(x-y)||f(y)|\,\mathrm{d}y.$$

In fact,we can easily see that when $\Omega\equiv 1$, $M_{\Omega,\alpha}$ is just the fractional maximal operator

$$M_\alpha f(x)=\sup_{Q\ni x}\frac{1}{|Q|^{1-\frac{\alpha}{n}}}\int_Q|f(y)|\,\mathrm{d}y,$$

especially in the limiting case $\alpha=0$, the fractional maximal operator reduces to the Hardy-Littlewood maximal operator.

It is well known that Calderón and Zygmund[9] have proven the fractional integral operator $T_{\Omega,\alpha}$ with a rough kernel is bounded on L^p. It turns out that such kind of operators are much more closely related to elliptic partial differential of second order with variable coefficients. For $0<\alpha<n$,Muchenhoupt and Wheeden[10] proved the boundedness of $T_{\Omega,\alpha}$ with power weights from L^p to L^q. In [11] Ding, Chen and Fan gave the boundedness properties of $T_{\Omega,\alpha}$ on Hardy Spaces. However, the corresponding results for $T_{\Omega,\alpha}$ have not been proven on variable exponent Lebesgue spaces. Similarly,Diening[2] discovered the Hardy-littlewood maximal operator is bounded on $L^{p(\cdot)}(\mathbf{R}^n)$, and Cruz-Uribe[3] investigated the boundedness of M_α on $L^{p(\cdot)}(R^n)$, but the boundedness of $M_{\Omega,\alpha}$ has not been studied.

Before stating our main results, let us recall some notations and definitions.

Definition 1. Suppose $p(\cdot):\mathbf{R}^n\to[1,\infty)$ is a measurable function for some $\lambda>0$, then the variable exponent Lebesgue space $L^{p(\cdot)}(\mathbf{R}^n)$ is defined by

$$L^{p(\cdot)}(\mathbf{R}^n)=\{f\ is\ \text{measurable}:\int_{\mathbf{R}^n}(|f(x)|/\lambda)^{p(x)}\mathrm{d}x<\infty\},$$

with the norm

$$\|f\|_{L^{p(\cdot)}(\mathbf{R}^n)}=\inf\{\lambda>0:\int_{\mathbf{R}^n}(|f(x)|/\lambda)^{p(x)}\mathrm{d}x\le 1\}.$$

We denote

$$p_-=\operatorname{ess\,inf}\{p(x):x\in\mathbf{R}^n\},\qquad p_+=\operatorname{ess\,sup}\{p(x):x\in\mathbf{R}^n\}.$$

Using this notation we define a class of variable exponent

$$\Phi(R^n)=\{p(\cdot):\mathbf{R}^n\to[1,\infty),\ p_->1,\ p_+<\infty\}.$$