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# REPRESENTATION OF GENERALIZED TRANSLATIONS OF THE PRODUCT OF TWO FUNCTIONS ON SIGNED HYPERGROUPS ON THE REAL LINE

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**Abstract.** In this paper, we consider the generalized translations associated with the Dunkl and the Jacobi-Dunkl differential-difference operators on the real line which provide the structure of signed hrpergroups on **R**. Especially, we study the representation of the generalized translations of the product of two functions for these signed hypergroups.

**Key words:** *Bessel function, Dunkl operator, generalized translation, signed hypergroup, Jacobi-Dunkl operator, Jacobi function*

**AMS (2010) subject classification:** 33C45, 33C10, 43A62, 42A38

## 1 Introduction

The generalized translation operators are introduced by Delsarte and Levitan (see [4], [9]) and an interesting harmonic analysis for them is developed, since they permit to define the convolution and this concept allows to introduce the so called hypergroups (see [2]) and signed hypergroups (see [13], [12]).

A natural question arises : how the translation of the product of two functions may be represented in the framework of signed hypergroups?

We begin by the following remark. If we consider even functions on **R**, the appropriate translation in this situation is given by

$$T_y f(x) = \frac{1}{2}[f(x+y) + f(x-y)].$$

In this case, we have

$$T_y(fg)(x) = T_yf(x)T_yg(x) + \frac{1}{4}[f(x+y) - f(x-y)][g(x+y) - g(x-y)]. \tag{1}$$

We observe that in contrast with the group situation, we do not have

$$T_y(fg)(x) = T_yf(x)T_yg(x),$$

and we remark the appearance of other terms. In [10] the authors have given analogous of the representation (1) in the context of Bessel-Kingman and Jacobi hypergroups on the half real line.

The aim of this paper is to extend analogous of the representation given in [10] in the context of signed hypergroups. More precisely, we are interested in establishing representations of $T_y(fg)$, where $T_y$ is the generalized translation operator defined in the Bessel-Dunkl and Jacobi-Dunkl signed hypergroups.

The structure of these signed hypergoups is derived from differential-difference operators on **R** of the form

$$\Lambda f(x) = f'(x) + \frac{A'(x)}{A(x)}(\frac{f(x) - f(-x)}{2}).$$

The Dunkl operator on **R** corresponds to the function $A(x) = |x|^{2\alpha+1}$, $\alpha > -\frac{1}{2}$. The Jacobi-Dunkl operator corresponds to the function $A(x) = 2^{2\rho}(\sinh|x|)^{2\alpha+1}(\cosh x)^{2\beta+1}$, $\rho = \alpha+\beta+1$.

M. Rösler[11], ( resp. N. Ben Salem and A. Ould Ahmed Salem[11], have established a product formula for the eigenfunctions of the Dunkl operator, ( resp. the Jacobi-Dunkl operator), which leads to generalized translations and uniformly bounded convolutions of point measures and generate structures of signed hypergroup on **R**.

The paper is organized as follows.

The first part is devoted to the study of the representation of the translation of product of two functions for the translation associated to the Dunkl operator.

The second part deals with a similar study for the translation associated to the Jacobi-Dunkl operator.

## 2 Representation of Translation of the Product of Two Functions for Bessel-Dunkl Signed Hypergroup

### 2.1 Preliminary