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QUARTIC CONVEX TYPE PIECEWISE POLYNOMIAL

Manprit Kaur and Arun Kumar
(*R. D. University, India)*

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Abstract. In the present paper we consider quartic piecewise polynomial for approximation to the function $f \in C^2[0,1]$. A convex type condition has been imposed in the partition so that the matrix involved for the computation of pp functions is of lower band. This reduces the computation for constructions of the pp functions for the approximation.

Key words: *piecewise polynomial, interpolation, deficient splines*

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1 Introduction and Notations

Let $0 = x_0 < x_1 < \cdots < x_n = 1$ be a mesh, denoted by Δ, of $[0,1]$. We write

$$x_i - x_{i-1} = h_i, \qquad i = 1, 2, \cdots, n$$

and π_m the set of all real algebraic polynomials of degree at most $m \geq 1$. When the partition points are equidistant i.e, uniform partition we write $h = h_i$, for i. The class of deficient polynomial splines of degree m with deficiency k, a non-negative integer, $k < m-1$ is defined as

$$S(m_k, \Delta) = \{s(x) : s(x) \in C^{m-k-1}[0,1], s(x) \in \pi_m, x \in [x_{i-1}, x_i], \qquad i = 1, 2, \cdots n\}.$$

For $k = 0$, i.e., $S(m_0, \triangle) = S(m, \triangle)$ denotes the class of splines of degree m.

The approximation by means of different kind of quartic spline functions has been studied by Marsden[5], Sharma and Tzimbalario[10] and Rana[8,9]. Spline functions specially cubic spline functions have been studied extensively, e.g. Meir and Sharma[6,7], Dikshit[1], Dikshit and Powar[2], and Kumar and Govil[4].

In order to reduce continuity requirements of the spline function, correspondingly restrictions of interpolations were imposed and such splines were termed as Deficient Splines. These functions are also termed as *pp* functions.

Here continuity requirement of the second derivative of splines function is replaced by the following condition:

$$\alpha s(x_{i-1} + m_1 h_i) + (1-\alpha) s(x_{i-1} + m_2 h_i) = s(x_{i-1} + h_i(\alpha m_1 + (1-\alpha) m_2)), \qquad i = 1, 2, \cdots, n, \tag{1}$$

where α, m_1 and m_2 are positive numbers such that $0 < m_1 < m_2 < 1$ and $0 < \alpha < 1$. We call this condition as a convexity type condition and denote such class of spline functions by $S(4_1^c, \Delta)$. $S(3_1^c, \Delta)$ has a similar meaning for cubic case which was studied by Kumar and Das in [3].

The spline in the class $S(4_1^c, \Delta)$ involves parameters α, m_1 and m_2. The approximation of the function naturally depends on the parameters, which are selected so that the error is minimum. We also consider deviation of the second derivative of the function and the spline function.

First we consider the existence of the Quartic Deficient Spline Function and prove the following:

Theorem 1. *Let α, m_1 and m_2 be non-negative real numbers such that $m_1 + m_2 = 1$, $\alpha \neq \{0, 1\}$ and $m_2 \neq \frac{1}{2}$. Then there exists a unique 1-periodic function of the class $S(4_1^c, \triangle)$ for uniform partition satisfying the interpolatory condition:*

$$f(x_i) = s(x_i), i = 1, 2, \cdots, n.$$

provided

$$m_2 \leq \frac{1}{\alpha^2 + (\alpha - 1)^2} \max\{\alpha^2, (\alpha - 1)^2\}$$
$$m_2 \geq \frac{1}{\alpha^2 + (\alpha - 1)^2} \min\{\alpha^2, (\alpha - 1)^2\}.$$

We make use of the following result:

Lemma . (a) *Let*

$$C_n(r, q, p; c_1, c_2) = \begin{bmatrix} q & p & 0 & 0 & . & . & . & c_1 \\ r & q & p & 0 & 0 & . & . & 0 \\ 0 & r & q & p & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ c_2 & 0 & 0 & . & . & . & r & q \end{bmatrix},$$

and $C_n = C_n(r, q, p; 0, 0) = C_n(r, q, p)$. We have

(i) $|C_n| = \dfrac{\beta_1^{n+1} - \beta_2^{n+1}}{\beta_1 - \beta_2}$, *where $\beta_1 + \beta_2 = q$ and $\beta_1 - \beta_2 = \sqrt{q^2 - 4pr}$;*

(ii) $|C_n(r, q, p; r, p)| = q|C_{n-1}| - 2pr|C_{n-2}| + (-1)^{n+1}(p^n + r^n)$,

where $|A|$ denotes the determinant of the matrix A.

(b) *If $C_n(r, q, p; r, p)$ is a non-singular matrix, then the (i, j) entry $\hat{a}_{ij}$ of its inverse matrix is given by*

$$\hat{a}_{ij} = \begin{cases} \dfrac{(-1)^{i-j}\{r^{i-j}|C_{n-(i-j)-1}| + (-1)^n p^{n-(i-j)}|C_{i-j-1}|\}}{|C_n(r,\ q,\ p;\ r,\ p)|}, & j < i, \\ \dfrac{(-1)^{j-i}\{p^{j-i}|C_{n-(j-i)-1}| + (-1)^n r^{n-(j-i)}|C_{j-i-1}|\}}{|C_n(r,\ q,\ p;\ r,\ p)|}, & j > i, \\ \dfrac{|C_{n-1}|}{|C_n(r,\ q,\ p;\ r,\ p)|}, & j = i, \end{cases}$$