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KY FAN TYPE BEST APPROXIMATION THEOREM FOR A CLASS OF FACTORIZABLE MULTIFUNCTIONS

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Abstract. An existence result on Ky Fan type best approximation is proved. For this purpose, a class of factorizable multifunctions and the other one being a demicontinuous, relative almost quasi-convex, onto function on an approximately weakly compact, convex subset of Hausdorff locally convex topological vector space are used. As consequence, this result extends the best approximation results of Basha and Veeramani[8] and many others.

Key words: *Almost affine, almost quasi convex, approximately weakly compact, best approximation, demicontinuous map, locally convex space, relative almost quasi convex, Kakutani factorizable multifunction, upper semicontinuous map*

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1 Introduction

The Theory of approximation is a mathematical study of how given quantities can be approximated by other (usually simpler) ones under appropriate conditions. Over the years, the theory has become so extensive that it intersects with every other branch of analysis at present. One of the central problems in approximation theory is to determine points that minimize distances to a given point or subset. The field of best approximation has dealt with this problem rigorously and it is an active field of research within Approximation theory. The best approximation has always attracted analysts because it carries enough potential to be extended especially with the functional analytic approach in nonlinear analysis. In the mid of the 20th century, it was found that existence of fixed point has its relevance in proving the existence of best approximation. The best approximation is termed as invariant approximation in the case of self mappings. It is important to mention here that when non-self mapping was considered, Ky Fan technique is found simply a wonderful tool to prove the existence of best approximation. This technique is hypothesis of Ky Fan theorem.

Ky Fan[4] Approximation theorem is as below:

Theorem 1.1.[4] *Let $\mathcal{C}$ be a non-empty compact convex subset of Hausdorff locally convex topological vector space $\mathcal{X}$ with a continuous semi-norm p and $\mathcal{T}:\mathcal{C}\to\mathcal{X}$ is a single valued continuous map, then there exists an element x_0, called a best approximant in $\mathcal{C}$, such that*

$$p(x_0-\mathcal{T}x_0)=d_p(\mathcal{T}x_0,\mathcal{C})=\inf\{p(\mathcal{T}x_0-y):y\in\mathcal{C}\}. \tag{1.1}$$

Initially, Ky Fan's approximation approach[4] helped in proving the existence of fixed point under different boundary conditions. Later, it was applied in the field of approximation theory, minimax theory, game theory, and variational inequality. During this phase, interesting extensions of Ky Fan's theorem were made and a variety of applications, mostly in fixed point theory and approximation theory, were given by many analysts (see e.g., [11]).

An interesting turn was given by Prolla[6] who extended Ky Fan Theorem by involving another mapping called almost affine mapping. Recently, interesting extensions of Ky Fan theorem 1.1 have been given by various authors for continuous multifunctions defined on noncompact convex subsets of a topological vector space possessing sufficiently many linear functionals.

Next, it was generalized by Reich[7] weakening the compactness as below:

Theorem 1.2.[7] *If $\mathcal{C}$ is a non-empty approximately $p-$compact, convex subset of a Hausdorff locally convex topological vector space $\mathcal{X}$ with a continuous semi-norm p and $\mathcal{T}:\mathcal{C}\to\mathcal{X}$ is a single valued continuous map with $\mathcal{T}(\mathcal{C})$ relatively compact, then there exists an element x_0 in $\mathcal{C}$ satisfying* (1.1) *of the Theorem* 1.1.

Next, Sehgal and Singh[9] generalized the result of Reich[7] for multi functions in normed linear spaces by generalizing the result of Prolla[6] as below:

Theorem 1.3.[10] *If $\mathcal{C}$ is a non-empty approximately compact convex subset of a normed linear space $\mathcal{X}$, $\mathcal{T}:\mathcal{C}\to\mathcal{X}$ a single valued continuous maps with $\mathcal{T}(\mathcal{C})$ relatively compact and $g:\mathcal{C}\to\mathcal{C}$ an affine, continuous, surjective single valued map such that g^{-1} sends compact subsets of $\mathcal{C}$ onto compact sets, then there exists an element x_0 in $\mathcal{C}$ such that $\|gx_0-\mathcal{T}x_0\|=d(\mathcal{T}x_0,\mathcal{C})$.*

Further generalization in this direction is done by Vetrivel et al[12]. who used Kakutani factorizable multifunctions in Hausdorff locally convex topological vector space. Carbone[2] replaced almost affine by almost quasi convex map and also extended the result of Prolla[6] by using approximately weakly compact subset of normed linear space[3].

Basha and Veeramani[8] also proved best approximation theorem for continuous Kakutani