Anal. Theory Appl.
Vol. 28, No. 1 (2012), 73–78

ABSOLUTE RETRACTIVITY OF SOME SETS TO TWO-VARIABLES MULTIFUNCTIONS

H. Afshari and Sh. Rezapour

(*Shahid Madani University, Iran)*

Received Nov. 12, 2011

Abstract. In this paper, by providing some different conditions respect to another works, we shall present two results on absolute retractivity of some sets related to some multifunctions of the form $F : X \times X \to P_{b,cl}(X)$, on complete metric spaces.

Key words: *absolute retract, fixed points set, multifunction*

AMS (2010) subject classification: 47H04, 47H10, 54H25

1 Introduction

In 1970, Schirmer provided some results about topological properties of the fixed point set of multifunctions[5]. Later, some authors continued this review by providing different conditions [1],[3]. Recently, Sintamarian proved some results on absolute retractivity of the common fixed points set of two multivalued operators[6],[7]. Also, Afshari, Rezapour and Shahzad proved some results about absolute retractivity of the common fixed points set of two multifunctions[4]. In this paper, by providing some different conditions respect to another works, we shall present two results on absolute retractivity of some sets related to some multifunctions of the form $F : X \times X \to P_{b,cl}(X)$. Let X and Y be nonempty sets, $P(Y)$ the set of all nonempty subsets of Y, and $F : X \to P(Y)$ a multifunctions. A mapping $\varphi : X \to Y$ is called a selection of F whenever $\varphi(x) \in Fx$ for all $x \in X$. Throughout the paper, for a topological space X we denote the set of all closed and bounded subsets of X by $P_{b,cl}(X)$ when X is a metric space.

Let (X,d) be a metric space, $B(x_0,r) = \{x \in X : d(x_0,x) < r\}$. For $x \in X$ and $A,B \subseteq X$, set $d(x,A) = \inf_{y\in A} d(x,y)$ and

$$H(A,B) = \max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\}.$$

It is known that, H is a metric on closed bounded subsets of X which is called the Hausdorff metric (for more details see [6] and [7]).

We say that a topological space X is an absolute retract for metric spaces whenever for each metric space Y, $A \in P_{cl}(Y)$ and continuous function $\psi : A \to X$, there exists a continuous function $\varphi : Y \to X$ such that $\varphi|_A = \psi$. Let $\mathcal{M}$ be the set of all metric spaces, $X \in \mathcal{M}$, $\mathcal{D} \in P(\mathcal{M})$ and $F : X \to P_{b,cl}(X)$ a lower semi-continuous multifunction. We say that F has the selection property with respect to $\mathcal{D}$ if for each $Y \in \mathcal{D}$, continuous function $f : Y \to X$ and continuous functional $g : Y \to (0,\infty)$ such that $G(y) := \overline{F(f(y)) \cap B(f(y), g(y))} \neq \emptyset$ for all $y \in Y$, $A \in P_{cl}(Y)$, every continuous selection $\psi : A \to X$ of $G|_A$ admits a continuous extension $\varphi : Y \to X$, which is a selection of G. If $\mathcal{D} = \mathcal{M}$, then we say that F has the selection property and we denote this by $F \in SP(X)$ (for more details see [6] and [7]).

2 Main Results

Theorem 2.1. *Let (X,d) be a complete metric space and absolute retract for metric spaces and $F : X \times X \to P_{b,cl}(X)$ a lower semicontinuous multifunction such that there exist $a_{11}, a_{12}, \cdots, a_{15}, a_{21}, a_{22}, \ldots, a_{25} \in (0,1)$ with $a_{11} + a_{13} + a_{14} + 2a_{12} < 1$, $a_{21} + a_{23} + a_{24} + 2a_{22} < 1$,*

$$\begin{aligned} H(F(u,v),F(x,y)) \leq\ & a_{11}d(x,u) + a_{12}d(x,F(u,v)) \\ & + a_{13}d(F(x,y),x) + a_{14}d(F(u,v),u) + a_{15}d(u,F(x,y)) \end{aligned}$$

and

$$\begin{aligned} H(F(u,v),F(x,y)) \leq\ & a_{21}d(y,v) + a_{22}d(y,F(u,v)) \\ & + a_{23}d(F(x,y),y) + a_{24}d(F(u,v),v) + a_{25}d(F(x,y),v) \end{aligned}$$

for all $u,v,x,y \in X$. Then the set $B = \{(x,y) : x,y \in F(x,y)\}$ is an absolute retract for metric spaces.

Proof. It is easy to see that $F \in SP(X \times X)$ and $X \times X$ is an absolute retract for metric spaces. Now, put $1 < q < \min\{(a_{11} + a_{13} + a_{14} + 2a_{12})^{-1}, (a_{21} + a_{23} + a_{24} + 2a_{22})^{-1}\}$ and

$$l := \max\{\frac{a_{11} + a_{12} + a_{13}}{1 - (a_{12} + a_{14})}, \frac{a_{21} + a_{22} + a_{23}}{1 - (a_{22} + a_{24})}\}.$$

It is not difficult to verify that $ql < 1$. Let Y be a metric space, $A \in P_{cl}(Y)$ and $\psi : A \to B$ a continuous function. Since $X \times X$ is an absolute retract for metric spaces, there exists a continuous function $\varphi_0 : Y \to X \times X$ such that $\varphi_0|_A = \psi$. Let $\varphi_0 = (\varphi_0^1, \varphi_0^2)$. Consider the function