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Approximation Properties by q-Durrmeyer-Stancu Operators

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Abstract. In this paper, we are dealing with q-Bernstein-Durrmeyer-Stancu operators. Firstly, we have estimated moments of these operators. Then we have discussed some approximation properties and asymptotic formulas. We have obtained better estimations by using King type approach and given statistical convergence for the operators.

Key Words: q-Durrmeyer operator, q-Jackson integral, q-Beta function.

AMS Subject Classifications: 41A25, 41A35

1 Introduction

We first mention some notations of q-calculus. Throughout the present article q is a real number satisfying the inequality $0<q\leq 1$. For $n\in\mathbb{N}$,

$$[n]_q=[n]:=\begin{cases} (1-q^n)/(1-q), & q\neq 1,\\ n, & q=1,\end{cases}$$

$$[n]_q!=[n]!:=\begin{cases} [n][n-1]\cdots[1], & n\geq 1,\\ 1, & n=0,\end{cases}$$

and

$$(1+x)_q^n:=\begin{cases} \prod\limits_{j=0}^{n-1}(1+q^jx), & n=1,2,\cdots,\\ 1, & n=0.\end{cases}$$

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For the integers n, k, $n\geq k\geq 0$, the q-polynomial coefficients are defined by

$$\left[\begin{array}{c} n \\ k \end{array}\right]=\frac{[n]!}{[k]![n-k]!}.$$

The q-analogue of integration, discovered by Thomae [16] and Jackson [10] in the interval $[0,a]$, is defined by

$$\int_0^a f(t)d_qt:=a(1-q)\sum_{n=0}^{\infty}f(aq^n)q^n,\quad 0<q\leq 1 \quad \text{and} \quad a>0.$$

In [3], the two q-Gamma functions are defined as

$$\Gamma_q(x)=\int_0^{1/1-q}t^{x-1}E_q(-qt)d_qt \quad \text{and} \quad \gamma_q^A(x)=\int_0^{\infty/A(1-q)}t^{x-1}e_q(-t)d_qt. \tag{1.1}$$

There are two q-analogues of the exponential function e^x, see [11],

$$e_q(x)=\sum_{k=0}^{\infty}\frac{x^k}{[k]!}=\frac{1}{(1-(1-q)z)_q^{\infty}},\quad |x|<\frac{1}{1-q},\quad |q|<1,$$

and

$$E_q(x)=\sum_{k=0}^{\infty}q^{k(k-1)/2}\frac{x^k}{[k]!}=(1+(1-q)x)_q^{\infty},\quad |q|<1.$$

By Thomae [16] and Jackson [10], it was shown that the q-Beta function defined by the usual formula

$$B_q(t,s)=\frac{\Gamma_q(s)\Gamma_q(t)}{\Gamma_q(s+t)}$$

has the q-integral representation, which is a q-analogue of Euler's formula:

$$B_q(t,s)=\int_0^1 x^{t-1}(1-qx)_q^{s-1}d_qx,\quad t,s>0. \tag{1.2}$$

Due to the importance of polynomials, a variety of their generalizations and related topics have been studied (see [5] and [15]). Recently, an intensive research has been conducted on operators based on q-integers. In 1997, G. M. Phillips [14] proposed the following q-analogue of the well-known Bernstein polynomials, for each positive integer n and $f\in C[0,1]$, are defined as

$$B_{n,q}(f;x):=\sum_{k=0}^{n}f\Big(\frac{[k]}{[n]}\Big)p_{n,k}(q;x), \tag{1.3}$$

where

$$p_{n,k}(q;x)=\left[\begin{array}{c} n \\ k \end{array}\right]x^k\prod_{s=0}^{n-k-1}(1-q^sx). \tag{1.4}$$