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All Meromorphic Solutions of Some Algebraic Differential Equations

Wenjun Yuan[1,2], Zifeng Huang[1], Jinchun Lai[1] and Jianming Qi[3,*]

[1] *School of Mathematics and Information Science, Guangzhou University, Guangzhou 510006, China*
[2] *Key Laboratory of Mathematics and Interdisciplinary Sciences of Guangdong Higher Education Institutes, Guangzhou University, Guangzhou 510006, China*
[3] *Department of Mathematics and Physics, Shanghai Dianji University, Shanghai 201306, China*

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Abstract. In this article, we introduce some results with respect to the integrality and exact solutions of some 2nd order algebraic DEs. We obtain the sufficient and necessary conditions of integrable and the general meromorphic solutions of these equations by the complex method, which improves the corresponding results obtained by many authors. Our results show that the complex method provides a powerful mathematical tool for solving a large number of nonlinear partial differential equations in mathematical physics.

Key Words: Differential equation, general solution, meromorphic function, elliptic function.

AMS Subject Classifications: 30D35, 34A05

1 Introduction

Nonlinear partial differential equations (NLPDEs) are widely used as models to describe many important dynamical systems in various fields of sciences, particularly in fluid mechanics, solid state physics, plasma physics and nonlinear optics Exact solutions of NLPDEs of mathematical physics have attracted significant interest in the literature Over the last years, much work has been done on the construction of exact solitary wave solutions and periodic wave solutions of nonlinear physical equations. Many methods have been developed by mathematicians and physicists to find special solutions of NLPDEs, such as the inverse scattering method [1], Darboux transformation method [2], Hirota

*Corresponding author. *Email addresses:* wjyuan1957@126.com (W. J. Yuan), 847071088@qq.com (Z. F. Huang), 190534202@qq.com (J. C. Lai), qijianmingsdju@163.com (J. M. Qi)

bilinear method [3], Lie group method [4], bifurcation method of dynamic systems [5–7], sine-cosine method [8], tanh-function method [9, 10], Fan-expansion method [11], and homogenous balance method [12]. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations. Recently, Kudryashov et al. [13–16] find exact meromorphic solutions for some nonlinear ordinary differential equations by using Laurent series and gave some basic results. Follow their work the complex method was introduced by Yuan et al. [17–19]. It is shown that the complex method provide a powerful mathematical tool for solving great many nonlinear partial differential equations in mathematical physics.

2 The second order algebraic differential equations with degree two

In 2013, Yuan et al. [17] derived all traveling wave exact solutions by using the complex method for a type of ordinary differential equations (ODEqs)

$$Aw''+Bw+Cw^2+D=0, \tag{2.1}$$

where A, B, C and D are arbitrary constants.

In order to state these results, we need some concepts and notations.

A meromorphic function $w(z)$ means that $w(z)$ is holomorphic in the complex plane $\mathbb{C}$ except for poles. α, b, c, c_i and c_{ij} are constants, which may be different from each other in different place. We say that a meromorphic function f belongs to the class W if f is an elliptic function, or a rational function of $e^{\alpha z}$, $\alpha\in\mathbb{C}$, or a rational function of z.

Theorem 2.1. *Suppose that $AC\neq 0$, then all meromorphic solutions w of an Eq.* (2.1) *belong to the class W. Furthermore, Eq.* (2.1) *has the following three forms of solutions:*

(I) The elliptic general solutions

$$w_{1d}(z)=-6\frac{A}{C}\Big\{-\wp(z)+\frac{1}{4}\Big[\frac{\wp'(z)+F}{\wp(z)-E}\Big]^2\Big\}+6\frac{AE}{C}-\frac{B}{2C}.$$

Here, $4DC=-12A^2g_2+B^2$, $F^2=4E^3-g_2E-g_3$, g_3 and E are arbitrary.

(II) The simply periodic solutions

$$w_{1s}(z)=-6\frac{A}{C}\alpha^2\coth^2\frac{\alpha}{2}(z-z_0)-\frac{A}{2C}\alpha^2-\frac{B}{2C},$$

where $4DC=-A^2\alpha^4+B^2$, $z_0\in\mathbb{C}$.

(III) The rational function solutions

$$w_{1r}(z)=-\frac{6\frac{A}{C}}{(z-z_0)^2}-\frac{B}{2C},$$

where $4CD=B^2$, $z_0\in\mathbb{C}$.