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Multiscale Basis Functions for Singular Perturbation on Adaptively Graded Meshes

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Abstract. We apply the multiscale basis functions for the singularly perturbed reaction-diffusion problem on adaptively graded meshes, which can provide a good balance between the numerical accuracy and computational cost. The multiscale space is built through standard finite element basis functions enriched with multiscale basis functions. The multiscale basis functions have abilities to capture originally perturbed information in the local problem, as a result our method is capable of reducing the boundary layer errors remarkably on graded meshes, where the layer-adapted meshes are generated by a given parameter. Through numerical experiments we demonstrate that the multiscale method can acquire second order convergence in the L^2 norm and first order convergence in the energy norm on graded meshes, which is independent of ε. In contrast with the conventional methods, our method is much more accurate and effective.

AMS subject classifications: 35J25, 65N12, 65N30

Key words: Multiscale basis functions, singular perturbation, boundary layer, adaptively graded meshes.

1 Introduction

Singularly perturbed problems have attracted much attention during the past decades. The perturbed parameters in the partial differential equations arise naturally or artificially. Its main difficulty lies in so-called boundary layer behavior, i.e., the solution varies rapidly in a thin boundary layer with a very small parameter ε. Using the standard finite element method (FEM) or finite difference method (FDM) to solve the problem directly

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is much costly and is not independently uniform-consistent. This motivates us to study efficient numerical methods for the singular perturbation problems (see [11, 13, 16]).

In recent years the numerical solutions of singularly perturbed problems have been intensively studied, and there are two major strategies. One is h refinement (h is mesh size) on layer-adapted meshes, e.g., [2, 12, 17, 19]. The other is p refinement (p is degree of approximating polynomials) or hp refinement (the combinations of h and p), e.g., [6, 18, 20]. Chen and Xu [3] presented a mathematical proof on accuracy and stability of the mesh adaptation for one dimensional singular problem. Shishkin [15] proposed a finite difference scheme on a priori adapted meshes for a singularly perturbed parabolic convection-diffusion model. Roos [14] considered a stabilized finite element method on layer-adapted meshes and applied the recovery techniques to acquire supercloseness results.

In addition, the finite element method can be extended to the multiscale scheme. For that purpose, Hou, Wu and Cai [7, 8] proposed the multiscale finite element method (MsFEM) by solving the local homogenization problem for basis functions, and provided convergence analyses and numerical examples for problems with rapidly oscillating coefficients. Araya and Valentin [1] considered the a posterior error estimates for the reaction-diffusion problem, and obtained consistent energy norm estimate. Efendiev and Hou [5] discussed the applications of MsFEM to two-phase immiscible flow simulation in which limited global information was taken into accounted, and the inverse problem was also discussed. Jiang and Huang [9] numerically investigated the MsFEM with rapidly oscillation coefficients and gave a good choice of boundary condition in the local problem for multiscale basis functions. On coarse uniform meshes Jiang and Sun [10] obtained much accurate results with the contributions of analytic singular basis functions to reduce the boundary layer errors remarkably. Efendiev, Galvis and Gildin [4] applied the spectral multiscale finite element method with the combination of local and global model reduction techniques, and achieved a balanced and optimal result in practical applications.

The new point in this paper is to demonstrate the accuracy and efficiency of multiscale basis functions combined with a modified version of graded meshes for singularly perturbed problems. The multiscale bases can capture the local boundary information on the layer-adaptively graded meshes, and therefore offer the uniform-consistent and predictably convergent solutions. When the conventional methods fail in cases, the proposed MsFEM is shown to obtain the accurate layer behaviors and reduce the computational costs, which may be applied in many realms.

The remaining part of this paper is organized as follows. In Section 2 we introduce the singularly perturbed reaction-diffusion model with small parameter ε and build the adaptively graded meshes for our MsFEM. In Section 3 we construct the enriched multiscale space through multiscale basis functions plus standard finite element basis functions. Numerical experiments are provided in Section 4, which demonstrate the efficiency and superiority of MsFEM on graded meshes for the singularly perturbed problem. And finally concluding remarks are given in Section 5.