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Partitioned Dashnic-Zusmanovich Type Matrices with Applications

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Abstract. We introduce a new subclass of H-matrices called partitioned Dashnic-Zusmanovich type (DZT) matrices and present the corresponding scaling matrices for this kind of matrices. There are three major applications. The first application is to provide equivalent eigenvalue localization related to index partition by using the nonsingularity of the new subclass. By taking some specific partitions, we provide other forms of eigenvalue localization sets that generalize and improve some well-known eigenvalue localization sets. The second application is to obtain an upper bound on the infinite norm of the inverse of partitioned DZT matrices using scaling matrices. The third application is to give an error bound of the linear complementarity problems (LCPs) by using scaling matrices. Additionally, we give another upper bound of the infinite norm and error bound of the LCPs by a reduction method, which transforms the given partitioned DZT matrix into the corresponding DZT matrix by partition and summation. The results obtained by the reduction method are generalizations of some known conclusions.

AMS subject classifications: 15A18, 15A42

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1. Introduction

Geometrically, n eigenvalues of an $n \times n$ complex matrix correspond to n points on the complex plane. Since solving the eigenvalues of a matrix is equivalent to finding the roots of the eigenpolynomial, and there is no root formula for polynomials with $n \geq 5$, iterative methods must be used to solve them. However, all iterative methods require an initial value. Therefore, knowledge of eigenvalues on the complex plane will accelerate the convergence of the iterative process.

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In addition, in many practical applications, it is often unnecessary to accurately calculate the eigenvalues of matrices but only to estimate their range. For example, in automatic control, it is necessary to estimate whether all eigenvalues of the matrix have negative real parts — i.e. whether they all are located in the left half-plane of the complex plane. In the stability theory of difference methods and iterative methods for solving linear systems, it is necessary to determine whether the eigenvalues of the matrix are all in the unit circle of the complex plane. Therefore, obtaining the eigenvalue localization set by simple calculations based on the elements of the original matrix is of great significance. The tighter the localization set given, the higher the localization accuracy will be.

The nonsingularity of the subclass of H-matrices is closely related to the eigenvalue localization problem. The most elegant example of this relation is the equivalence between strictly diagonally dominant (SDD) matrices and the well-known Geršgorin theorem [29]. For more results, please refer to [7, 29]. Therefore, exploring the broader subclasses of H-matrices is of great significance. Recently, a meaningful subclass of H-matrices called Dashnic-Zusmanovich type (DZT) matrices has been proposed in [31]. We wondered if it is possible to generalize the DZT matrix to a larger range using the idea of partitioning while retaining its H-matrix property. The answer is positive.

In this work, we propose partitioned Dashnic-Zusmanovich type matrices, which turn out to be a subclass of H-matrices. This class of matrices contains many known subclasses of H-matrices. Based on the nonsingularity of partitioned DZT matrices, we give a tighter eigenvalue localization. On one hand, we present a way to construct the scaling matrices of partitioned DZT matrices. Using the scaling matrices of partitioned DZT matrices, we discuss the applications of such matrices to the upper bound of infinite norm of inverse matrix and the linear complementarity problems (LCPs). On the other hand, we give another upper bound of the infinite norm and error bound of the LCPs by a reduction method that allows us to turn the problem into computing the corresponding DZT matrix through partition and summation. Many of the results in this paper are generalizations, improvements, and supplements to existing conclusions.

The present paper is structured as follows. Section 2 introduces partition DZT matrices and proves that it is a subclass of H-matrices by using scaling matrix. Section 3 shows that the new matrix classes contain many known subclasses of H-matrices. The eigenvalue localization equivalent to the new matrix class is presented in Section 4. The applications of the idea of scaling matrices of new matrix classes to the estimation of the infinite norm of the inverse matrix and the LCP are discussed in Sections 5 and 6, respectively. In Section 7, a reduction method is applied to the infinite norm bound estimation and the error bound of the LCP associated with such matrices.

2. $\langle N\rangle$-DZT and Partitioned DZT Matrices

2.1. Definition and properties of $\langle N\rangle$-DZT matrices

Let $n \geq 2$ be a positive integer and $[n] = \{1, 2, \dots, n\}$. Set