

## Orthogonality Sampling Method for Identifying Small Anomalies in Real-World Microwave Imaging

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**Abstract.** In this paper, the application of the orthogonality sampling method (OSM) to the real-world microwave imaging for identifying location of small anomalies is addressed. In order to show the feasibility and limitation of the OSM, we theoretically prove that the indicator function can be represented in terms of an infinite series of Bessel functions of integer order and the transmitting and receiving signal antenna configurations. This is based on the application of the Born approximation and the reciprocity of the incident fields. Throughout real-data experiments, it was shown that the OSM works well for identifying single anomaly under the specific location of transmitter while further improvement is needed for identification of multiple anomalies. To improve the imaging performance, we consider traditional indicator function with multiple sources and design a new indicator function with multiple sources weighted by the incident field. Theoretical results are contained to demonstrate the improvement.

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### 1. Introduction

Orthogonality sampling method (OSM) is a non-iterative technique to identify the locations of small scatterers or to produce the images of arbitrary shaped targets in inverse

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scattering problem. From the beginning of the pioneering research [30], OSM has been applied to various inverse scattering problems for example, multi-frequency OSM for identifying impenetrable objects [13], three-dimensional electromagnetic inverse scattering problem [15], identification of arbitrary shaped penetrable objects [8] and short, sound-soft open arcs [2]. Thanks to the referred contributions, it has been confirmed that the OSM is very fast to execution, robust against to the random noise, and simple to apply because it does not require any additional operations, e.g., singular-value decomposition, solving ill-posed integral equations or adjoint problems.

Inspired by these positive factors, the OSM has been recently extended to the various problems highly related to the microwave imaging for example, near-field orthogonality sampling method [6], improved near-field orthogonality sampling method for qualitative microwave imaging [5], cross-borehole ground penetrating radar (GPR) imaging [4], and anomaly identification from measured scattering parameters with single source [9]. Based on the remarkable contributions, OSM will contain many potential applications in microwave imaging for identifying locations or shapes of unknown anomalies.

In this contribution, we apply the OSM for a real-world microwave imaging for identifying locations of small anomalies from collected scattered field  $S$ -parameters with single and multiple sources. For this purpose, we design an indicator function of the OSM on the basis of the integral representation formula for the scattered field  $S$ -parameters in the existence of small anomaly. Notice that opposite to the recent work [9], it is very hard to collect the measurement data when the location of an antenna which transmits and receives the signal so that the structure of designed indicator function is different from the traditional one. In order to explain some intrinsic attributes, feasibilities and limitations of identification, we establish a mathematical theory of the indicator function of the OSM by proving that it is expressed by an infinite series of Bessel function and antenna configuration. We then illustrate some experimental results to demonstrate the theoretical results and to show the effectiveness and limitation.

For a further improvement, we consider the indicator function of the OSM with multiple sources; the main idea was proposed recently in the context [30]. To show the feasibility and improvement of the traditional indicator function with multiple sources, a stability analysis is provided and experimental results are shown. Furthermore, we also design an indicator function with multiple sources weighted by the incident field for another improvement. We also provide mathematical theory about the feasibility and improvement of the indicator function as well as the unique determination of anomaly, and experimental results to demonstrate the improvement.

This paper is organized as follows. In Section 2, basic concept of the scattered field  $S$ -parameter is discussed and the indicator function of the OSM with single source is designed. Theoretical result of the designed indicator function and discussion about the OSM in real-world microwave imaging are contained in Section 3. Then we present the experimental results with real-data in Section 4 for various anomalies for showing both the feasibilities and limitations of the OSM. In Section 5, we introduce the typical indicator function of the OSM with multiple sources, provide the stability analysis, and exhibit experimental results for a better imaging performance. Next, we design a new indicator function with multiple