Journal of Computational Mathematics
Vol.42, No.4, 2024, 1080–1108.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2301-m2022-0091

A NONLOCAL KRONECKER-BASIS-REPRESENTATION METHOD FOR LOW-DOSE CT SINOGRAM RECOVERY*

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Abstract

Low-dose computed tomography (LDCT) contains the mixed noise of Poisson and Gaussian, which makes the image reconstruction a challenging task. In order to describe the statistical characteristics of the mixed noise, we adopt the sinogram preprocessing as a standard maximum a posteriori (MAP). Based on the fact that the sinogram of LDCT has nonlocal self-similarity property, it exhibits low-rank characteristics. The conventional way of solving the low-rank problem is implemented in matrix forms, and ignores the correlations among similar patch groups. To avoid this issue, we make use of a nonlocal Kronecker-Basis-Representation (KBR) method to depict the low-rank problem. A new denoising model, which consists of the sinogram preprocessing for data fidelity and the nonlocal KBR term, is developed in this work. The proposed denoising model can better illustrate the generative mechanism of the mixed noise and the prior knowledge of the LDCT. Numerical results show that the proposed denoising model outperforms the state-of-the-art algorithms in terms of peak-signal-to-noise ratio (PSNR), feature similarity (FSIM), and normalized mean square error (NMSE).

Mathematics subject classification: 92C55, 68U10, 65K05.
Key words: Low-dose computed tomography, Kronecker-basis-representation, Low-rank approximation, Noise-generating-mechanism.

* Received April 16, 2022 / Revised version received October 8, 2022 / Accepted January 11, 2023 / Published online September 6, 2023 /
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1. Introduction

The computed tomography (CT) reconstruction is one of the most popular topics in medical image processing, especially for the low-dose CT. In practice, the CT scan is usually implemented in a low-dose way for reducing radiation harm to patients, and thus more noise is generated in CT images. Generally speaking, LDCT mainly contains the mixed statistical and electronic noise. The statistical noise stems from the fluctuations in detecting a finite number of X-ray quanta, and the electronic noise originates from the received analog signals contaminated by some noise as illustrated by [23,33]. Therefore, we can assume that the statistical noise obey Poisson distribution, and the electronic noise obey Gaussian distribution.

In order to effectively remove the mixed noise, many methods were proposed to reconstruct the LDCT. There are mainly three types of methods to improve the quality of low-dose CT images: methods based on iterative reconstruction, reconstruction based on deep learning, and reconstruction based on sinogram filtering. For the iterative reconstruction, various optimization methods are used to iteratively reconstruct the LDCT [11, 12]. Specifically, the iterative method is to denoise the low-dose CT in image domain by utilizing classical denoising methods. For example, Li *et al.* [21] proposed a modified nonlocal mean (NLM), which can adapt to the noise level of the LDCT image. Chen *et al.* [7] proposed an improved block-matching and 3D filtering (BM3D) method based on the context to reduce the noise in LDCT. Chen *et al.* [8] exploited a patch-based dictionary learning method for improving abdomen tumor LDCT images. By making full use of the self-similarity, Jia *et al.* [17] proposed a discriminative weighted nuclear norm minimization (DWNNM) for the LDCT, which improves the accuracy of block matching. For deep learning (DL) based methods, the principle is to make use of many CT image data to derive the reconstruction of LDCT. Some classical and up-to-date DL based methods have been proposed, Chen *et al.* [5] proposed a residual encoder-decoder convolutional neural network (RED-CNN) model by incorporating a deconvolution network and shortcut connections into a CNN model. In addition, He *et al.* [15] proposed a DL-based strategy for the model-based iterative reconstruction (MBIR), which can simultaneously handle the prior knowledge design and the MBIR parameter selection in one optimization framework. In order to avoid the limitation for supervised methods which require the low-dose and high-dose CT image pairs and the end-to-end pattern, Zeng *et al.* [41] developed a noise-generating-mechanism-driven unsupervised DL network for low-dose CT image. To overcome the limitation, Tao *et al.* [27] proposed an effective deep neural network that uses the slice of view-by-view back projection tensor as input and the image as output. For the reconstruction based on sinogram filtering, which is a preprocessing method for the raw sinogram data obtained by the CT scanner, many classical methods were proposed to restore the sinogram data of LDCT. For example, La Rivière *et al.* [19, 20] proposed a sinogram smoothing method to estimate the line integral of LDCT by maximizing a penalized-likelihood (PL) objective function, and use PL method to restore the sinogram of CT. Furthermore, Wang *et al.* [32] presented to use the penalized weighted least-squares (PWLS) approach to reduce the sinogram noise, and developed three different PWLS-based methods to remove the sinogram noise. Moreover, Gao *et al.* [13] proposed a two-step image reconstruction strategy which firstly adopts an adaptive sinogram restoration method, and then makes use of the total variation based projection to convex sets (POCS-TV) method to reconstruct the LDCT. In order to better depict the mixed noise, Xie *et al.* [37] proposed to make a low-dose CT sinogram preprocessing as an improved maximum a posteriori (IMAP) estimation, and take the first/second-order total variation as prior term