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A Buckley-Leverett Theory Based Lattice Boltzmann Method for Immiscible Two-Phase Flow with Viscous Coupling in Porous Media

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Abstract. In this paper, a lattice Boltzmann method is developed to simulate the water displacement of oil in porous media at the macroscopic-scale, which is built upon the generalized Darcy's law while incorporating cross terms to consider the viscous coupling between two phases. The Buckley-Leverett equation is applied to describe the saturation front advance, thus allowing the determination of water saturation at shock front by using the Welge's graphic method. We explore the effect of viscous coupling on the displacement by comparing the positions of the shock front under the conditions with different degrees of coupling and without viscous coupling. Results show that the advancing speed of shock front increases with the degree of coupling when the viscosity ratio of oil to water remains at 5. We also find that the effect of viscous coupling on the displacement can be neglected when the tolerance quantifying the degree of coupling equals the reciprocal of viscosity ratio. In addition, the effect of viscous coupling on the displacement is very sensitive to the change of viscosity ratio, which decreases monotonously with the decrease of viscosity ratio under the same coupling degree.

AMS subject classifications: 76S05, 76T06

Key words: Two-phase flows in porous media, explicit coupling terms, Buckley-Leverett theory, lattice Boltzmann method, viscosity ratio.

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1 Introduction

The simulation of immiscible two-phase flows in porous media has received wide attention in both scientific and industrial fields, such as groundwater flow, subsurface systems for CO2 sequestration, chemical engineering, nuclear safety, and petroleum engineering [1–5]. Currently, numerical simulation of two-phase flows in porous media can be classified into two main types: pore-scale simulation and macroscopic-scale (or continuum-scale) simulation. In the past few decades, the pore-scale simulation of two-phase flows in porous media has gained tremendous development with the advancement of computer power, high-performance algorithms and imaging techniques such as X-ray microtomography and nuclear magnetic resonance imaging [6–8]. The methods for pore-scale simulation of two-phase flows include but not limited to the volume-of-fluid method [9], the level-set method [10], the phase-field method [11], which solve the governing equations for mass conservation, momentum conservation, and interface evolution in realistic and accurate microscopic pore structures obtained from direct experimental imaging and computer reconstruction. However, pore-scale simulations usually require massive computational resources, which makes it impossible to simulate reservoir-scale problems [7]. In contrast, macroscopic-scale simulations have been widely applied to explore and comprehend two-phase flows in oil and gas engineering [12]. The macroscopic-scale models ignore the non-homogeneity of the microstructure in porous media, and use the macroscopic conservation equations derived on the basis of the volumetric averaging theories in representative elementary volumes. Besides, the parameters such as porosity, permeability, specific surface area, and pore size may also be introduced to account for the microstructure of porous media [7, 13].

In practice, macroscopic-scale two-phase flows in porous media are often described by simply generalizing the single-phase Darcy's law while introducing the concept of permeability to describe the flow capacity of each phase [14]. Despite their widespread use [15–18], the macroscopic-scale models corresponding to the generalized Darcy's law still suffer from the criticism for the lack of considering viscous coupling. This viscous coupling between phases could be considered by introducing cross terms into the generalized Darcy's law. The terminology "inter-phase viscous coupling" refers to the effect of viscous drag that one fluid motion in a pore may have on an adjacent immiscible fluid near the fluid-fluid interface that separates different immiscible fluids in the pore [19]. In the pioneering studies, Whitaker [20, 21] applied the volume averaging method to the analysis of immiscible two-phase flows in porous media and derived a model with a closure using the effective parameters containing coupling effects. In combination with the order analysis, it was shown that these nontraditional explicit coupling terms may play an important role in the flows with low fluid volume fraction. By investigating the flow in a porous medium with ideal tubes and cracked pore spaces as prototypes, Rose [22] qualitatively showed the importance of considering coupling effects, and for the first time gave the values of some transport coefficients in the equations describing fluid flows when coupling effects are taken into account. Kalaydjian et al. [23] investi-