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A New Unified Form of Universal Wall Function

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Abstract. A new form of universal wall function is proposed in this work, which can be applied to a variety of eddy viscosity models including one-equation models and two-equation models. The new wall function approximates the analytical results of the Spalart-Allmaras turbulence model in the near wall region of the zero pressure gradient incompressible turbulent boundary layer. This approximate form can be applied as a unified eddy-viscosity-based wall function and a concise unified wall function of velocity distribution is proposed after integration. On this basis, the grid Reynolds number is introduced and a function for judging the position of the first layer grid is given. The new wall function is validated using the NACA0012 and the axisymmetric separated boundary layer. It is proved that the wall function is accurate and universal, and the grid requirements are lower in applications.

AMS subject classifications: 65M10, 78A48

Key words: Eddy viscosity models, wall function, zero pressure gradient.

1 Introduction

Although computational capacity has improved rapidly in recent decades, the enormous computational cost is still a fundamental factor limiting the application of direct numerical simulations (DNS) and large eddy simulations (LES) in computational fluid dynamics [1]. To obtain accurate friction coefficients and heat flow, we have to set massive grids in the near wall region [2, 3], even using turbulence simulation methods with less computational cost, such as eddy viscosity models. In the near wall region, the flow gradient is very large and the effects of turbulent production, transport, dissipation, and diffusion are the strongest [4, 5]. This makes the treatment methods have a great impact on the accuracy of turbulence simulation in the near wall region. The treatment for the near wall region mainly includes the low-Reynolds-number model and wall function now. Compared with the low-Reynolds-number model with large grid requirements and arbitrary

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coefficient adjustment, the wall function is simple, economical, and robust [1]. Therefore, the further improvement and generalization of the wall function become urgent and important.

The wall function is a method that uses the supposed near-wall region velocity distribution or eddy viscosity coefficient distribution instead of the non-slip boundary condition. It sets the first layer grid in the log-law region, thus abandoning the calculation of the log-law region and below areas [6,7]. The application of the wall function is based on the following facts: for incompressible flow, there exist similar solutions in locally equilibrious boundary layer from the wall to log-law region. Since the wall function originally used is the log-law [8], it is only a universal log-law region solution. With the continuous application of the wall function, people began to emphasize the consistency and effectiveness of the whole near wall region. Evidently, a unified formula is not only more mathematically simple, but also more convenient to use than the two-layer or three-layer models.

For the zero pressure gradient turbulent boundary layer, Reichardt [9] first gave the unified wall function relation that can be used in the viscous sublayer, buffer layer, and log-law region, but the formula form is complex. van Driest [10] modified the existing method by introducing the damping function in the mixing length, thus obtaining a more accurate unified wall function. However, this wall funtion needs integral solutions in application. Splading [11] transformed the usual velocity distribution wall function $U^+ = f(y^+)$ to $y^+ = f(U^+)$. So a sufficiently concise unified wall function that is widely used was proposed based on Reichardt's near wall theory [9]. In recent years, Allmaras et al. [12] And Musker [13] also proposed unified wall functions on the basis of previous studies. For the case of adverse pressure gradient, it is necessary to introduce additional velocity scaling u_p in the wall function, which leads to another form when $u_p = 0$ [5,14]. Shih et al. [15] gave a wall function based on Tenekes's research findings [5] to solve this problem, and it can be directly applied to the zero pressure gradient case without changing the formula form.

In addition, in actual calculations, grid generation can't determine the specific position where each grid point in the first layer grid is located in the turbulent boundary layer in advance. So we introduce the Reynolds number of the first layer grid into the wall function, i.e., $Re_y = U_1 y_1/\nu = U^+ y^+$ [16]. Besides, it's permanent that $y^+ = \sqrt{Re_y}$ under the wall law of $U^+ = y^+$. The function $y^+ = \sqrt{Re_y}$ can be used to judge the position of turbulent boundary layer in the viscous sublayer. Furthermore, by introducing the Reynolds number of the first layer grid into the wall function, we can obtain a function that can be applied in a wider range. Aleksandar Jemcov [16] improved the wall function proposed by Shih et al. [15] through this method. Although researchers point out that such an improvement can avoid the iterative calculation of wall friction velocity [16], the method to further judge the position of the first layer grid is not given.

This paper aims to give a unified wall function of typical velocity distribution suitable for zero pressure gradient incompressible turbulent boundary layers. The velocity distribution relationship is simple and without integral, which gives the form of $U^+ = f(y^+)$