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A Fast Implementation of the Linear Bond-Based Peridynamic Beam Model

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Abstract. While the theory of peridynamics (PD) holds significant potential in engineering, its application is often limited by the significant computational costs by the nonlocality of PD. This research is based on a three-dimensional(3D) complex Timoshenko beam structure with six degrees of freedom. We propose a fast meshfree method based on the linear bond-based PD model of the stiffness matrix structure by ingeniously using the matrix decomposition strategy to maintain the Teoplitz structure of the stiffness matrix. This method significantly reduces the amount of calculation and storage without losing accuracy, reduces the amount of calculation from $\mathcal{O}(N^2)$ to $\mathcal{O}(N\log N)$, and decreases the storage capacity from $\mathcal{O}(N^2)$ to $\mathcal{O}(N)$. We validate the effectiveness of our approach through numerical examples, particularly in multi-beam structures. We demonstrate that our method realizes algorithm acceleration in numerical simulations of multi-beam structures subjected to static concentrated loads.

AMS subject classifications: 65R20, 65Y10

Key words: Bond-based peridynamics, beam model, fast methods, Teoplitz matrix.

1 Introduction

Classical continuum mechanics relies on partial differential equations, making it challenging to describe models with discontinuities. To address this issue, peridynamics was proposed by Silling [1] as a nonlocal model based on an integral equation. This model effectively solves the problems of crack propagation and multiple crack propagation [2]. Consequently, peridynamics has attracted extensive research in modeling methods, digital technology, and applications. Three constitutive relations have been defined for PD [3]: bond-based PD, ordinary state-based PD, and nonordinary state-based

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PD. While the bond-based PD limits Poisson's ratio to a fixed value, this restriction is relieved in the state-based model. Currently, PD is widely used to simulate damage prediction and crack propagation in various materials, including elastic and plastic ones [4–7], as well as nonlinear elastic and composite materials [8–11].

Analyzing complex engineering structures using three-dimensional PD models can be computationally expensive. To mitigate this, structure-simplified models such as beams, plates, and shells are often used to reduce the computational cost [12,13]. Silling et al. first introduced a one-dimensional bar model with axial force [14]. For the Euler-Bernoulli beam, both ordinary and non-ordinary state-based PD beam models have been proposed [15,16]. More recently, a three-dimensional Euler-Bernoulli PD beam model was proposed by Liu et al. [17]. For the Timoshenko beam, a PD beam model that describes axial, bending, and torsional deformations has been introduced [18]. Other models, such as those proposed by Diyaroglu et al. that introduce shear deformations using a bond-based PD beam model [19], and a 6 DOFs bond-based PD beam model that includes axial, bending, shear, and torsional deformation [20], have also been proposed. Moreover, the mixed formulation method has been developed to alleviate shear locking in PD beam models [21]. Yang et al. extended the PD beam and plate models to finite element frames [22], proposing high-order beam and functionally graded Timoshenko beam formulations [23,24]. Recently, PD beam and shell models based on the micro-beam bond were proposed using the via interpolation method [25,26].

Numerous numerical methods, such as meshfree, finite difference, finite element, and collocation methods, have been developed to solve the above PD models [27–33]. The asymptotic compatibility scheme, which preserves the limit behavior of the zero horizons of the non-local operator into the limit behavior of the local differential operator, provides consistency between the local and non-local models [33,34]. However, the high computational cost of PD poses limitations, particularly for multidimensional situations, even in the case of structure-simplified PD models. To overcome this issue, several efforts have been made, such as coupling methods that use PD only in the area around the crack while using classical mechanics [35,36] and other geometric analyses [37,38] in other areas. Additionally, fast methods based on the convolution structure of the PD model have been proposed [39,40]. To reduce the computational expense, a super-fast peridynamic model that reduces the number of inner loop operations has been introduced [41].

Recently, researchers have turned to the use of stiffness matrix structures as a way of reducing computational costs without sacrificing accuracy, a trend popularized by faster methods that decrease calculation times from $\mathcal{O}(N^2)$ to $\mathcal{O}(N\log N)$. In 2010, a fast method [42] on the basis of the stiffness matrix Toeplitz structure was proposed to solve the one-dimensional static linear bond-based peridynamics. Then, according to the two-dimensional nonlocal diffusion model [43], a fast collocation method based on the TBT matrix structure was given, which can be considered an approximate scalar-valued model. And a fast collocation method of two-dimensional static linear bond-based peridynamics with volume boundary conditions was found [44]. We used an equivalent but more effective method to evaluate it. A fast method [45] was also proposed to solve