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Analysis of Weakly Nonlinear Evolution Characteristics of Flow in the Constant Curvature Bend

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Abstract. The meandering river is an unstable system with the characteristic of nonlinearity, which results from the instability of the flow and boundary. Focusing on the hydrodynamic nonlinearity of the bend, we use the weakly nonlinear theory and perturbation method to construct the nonlinear evolution equations of the disturbance amplitude and disturbance phase of two-dimensional flow in meandering bend. The influence of the curvature, *Re* and the disturbance wave number on the evolution of disturbance amplitude and disturbance phase are analyzed. Then, the spatial and temporal evolution of the disturbance vorticity is expounded. The research results show: that the curvature makes the flow more stable; that in the evolution of the disturbance amplitude effected by curvature, *Re* and the disturbance wave number, exist nonlinear attenuation with damping disturbances, and nonlinear explosive growth with positive disturbances; that the asymmetry distribution of the disturbance velocities increases with the curvature; that the location of the disturbance vorticity's core area changes periodically with disturbance phase, and the disturbance vorticity gradually attenuates/increases with the decrease of the disturbance phase in the evolution process of damping/positive disturbances. These results shed light on the construction of the interaction model of hydrodynamic nonlinearity and geometric nonlinearity of bed.

AMS subject classifications: 76E30, 34C60

Key words: Curvature bend, hydrodynamics, weakly nonlinearity, disturbance vorticity.

1 Introduction

Since the 1980s, scholars have kept investigating the characteristics of the stability and nonlinearity of meandering rivers. Callander [1] considered that the instability is the

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cause of the river bending or braiding. In the instability analysis by Ikeda, Parker and Sawai [2], the most unstable wavelength was considered as the finite amplitude wavelength of the meandering river, which assumed the curvature ratio (the ratio of the half width to the curvature radius) is far less than 1. Parker, Sawai and Ikeda [3] analyzed the geometrical nonlinear stability of meandering river, ignoring the nonlinear dynamic terms. In the weakly nonlinear analysis of the meandering river by Seminara and Tubino [4], the geometric weakly nonlinear analysis was carried out near the resonance state, and the suppression of the nonlinear effect was revealed. Imran, Parker and Pirmez [5] pointed out that the study of Sun [6] has a fundamental defect in simulating the evolution of the meandering river because the nonlinear effect of flow dynamic is ignored. The meandering river has the instability mechanism, which refers to the instability of the flow [7]. Pittaluga, Nobile and Seminara [8] thought that the linear theory of the meandering river explains the resonance mechanism. However, the complete nonlinear theory of the meandering river has not been established yet, and the nonlinear effect has a certain influence on the flow field. Pittaluga and Seminara [9] argued that nonlinearity and instability are the important characteristics of the meandering river, while the effects of nonlinearity have been seriously ignored. Bai et al. [10–12] suggested that the nonlinear hydrodynamics theory is important to investigate the evolution of rivers under disturbances. Nelson, Pittaluga and Seminara [13] presented a nonlinear asymptotic theory of fully developed flow and bed topography subjected to unerodible bedrock layer, but they ignored the nonlinearity of the flow itself.

The previous studies mainly focused on the geometric nonlinearity of the meandering river and the bed disturbance. However, the study on the characteristics of the nonlinearity and evolution of the flow dynamics in the meandering river with different curvatures is insufficient. Different from the weakly nonlinear instability in the plane Poiseuille flow and shear layers [14, 15], in this paper, the instability and nonlinear evolution of the flow dynamics affected by the curved boundary with damping and positive disturbances are analyzed by constructing the control equation under the small flow disturbance. Under the conditions of the time mode, the Orr-Sommerfeld equation is used to analyze the stability characteristics of the hydrodynamic in the bend. And the Landau-Stuart equations are used with the weakly nonlinear theory of flow stability [16, 17]. Then the nonlinear evolution equations of the hydrodynamic of the constant curvature bend under the nonlinear effect of the flow are derived. The influences of the material composition of the river bank and the form of the bed surface are ignored. Generally, the river's curvature is constant along the bend, and the width is limited by the walls, such as in the canyon channel of Jing River in Shaanxi Province, China (Fig. 1). And in order to reduce the complexity of equations, we take the constant curvature form to investigate the stability and nonlinearity of hydrodynamic in the meandering river. The hydrodynamic study in the constant curvature bend provides the nonlinear hydrodynamic basis for further exploring the complete nonlinear relationship between river hydrodynamic and bed morphology. And in natural rivers, a high-order method is an accurate tool to study three-dimensional hydrodynamics [18].