Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 5, pp. 1335-1378

DOI: 10.4208/aamm.OA-2022-0113
October 2023

A Well-Balanced FVC Scheme for 2D Shallow Water Flows on Unstructured Triangular Meshes

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Received 20 April 2022; Accepted (in revised version) 8 February 2023

Abstract. This paper aims to present a new well-balanced, accurate and fast finite volume scheme on unstructured grids to solve hyperbolic conservation laws. It is a scheme that combines both finite volume approach and characteristic method. In this study, we consider a shallow water system with Coriolis effect and bottom friction stresses where this new Finite Volume Characteristics (FVC) scheme has been applied. The physical and mathematical properties of the system, including the C-property, have been well preserved.

First, we developed this approach by preserving the advantages of the finite volume discretization such as conservation property and the method of characteristics, in order to avoid Riemann solvers and to enhance the accuracy without any complexity of the MUSCL reconstruction. Afterward, a discretization was applied to the bottom source term that leads to a well-balanced scheme satisfying the steady-state condition of still water. A semi-implicit treatment will also be presented in this study to avoid stability problems due to source terms. Finally, the proposed finite volume method is verified on several benchmark tests and shows good agreement with analytical solutions and experimental results; moreover, it gives a noteworthy accuracy and rapidity improvement compared to the original approaches.

AMS subject classifications: 65M08, 35L65, 76M12

Key words: Shallow water model, method of characteristics, FVC scheme, finite volume method, well-balanced scheme.

1 Introduction

Water is a crucial issue for poverty reduction, sustainable development, and achieving the Millennium Development Goals. However, until now, some 2.1 billion peo-

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ple, or 30% of the world's population, still do not have access to a safe water supply (`www.who.int/news/18-06-2019`). Climate change and uncontrollable human activities cause flood accidents to become more frequent in recent decades. As a result, integrated water resources management is necessary and even indispensable for preventing floods and droughts. The preservation of the environment, the prevention, and control of the impact of natural risks are at the heart of major socio-economic issues.

In water resources management, numerical modelling is still an essential tool, and we can cite numerous numerical modelling applications of free surface flows to the management of water resources, environmental and ecosystem protection: simulation of flows due to a dam break, diversions of floods from a river to a water retention area, the change process of a river bed simulation, sediment or pollutant transport simulation in estuary and coastal environments, (see e.g., [1–5]). In rivers, estuaries and coastal areas, flows are characterized by: great topographical and morphological complexity, strong affection, or even pure advection in the case of a dam break on a flat and slippery bottom (without friction), a variable space scale (starting from a dozen to a few thousand meters) and in time scale (starting from a few minutes to several days).

Consequently, when developing a numerical approach to solve a free surface flow or other models, one encounters major difficulties which result from the physical complexity of the area and numerical calculations. We consider in this paper a derivative as a formal first-order approximation of the three-dimensional free surface incompressible Navier–Stokes equations, using the so-called shallow-water assumption. The difficulty in defining accurate numerical schemes for such hyperbolic systems is related to their nonlinear behaviour or, more generally, mathematical structure and the physical phenomena they generate. In particular, the presence of a shock front essentially causes numerical oscillations or artificial scattering, which are due to the treatment of the advection terms in the equations governing the transport of the water mass by a standard method of approximation. Another fundamental point is to get schemes that satisfy the preservation of steady states, such as still water equilibrium in the context of the shallow water system. Different approaches to satisfy the well-balanced property have been proposed (see, e.g., [3, 6–11]) and recent extensions to other types of homogeneous solvers can be found in [12–14]. The existence of non-trivial stationary states, i.e., for which the unknowns are not constant on the domain, is one of the specificities of the shallow water system, linked to the presence of source terms. This question has been an important subject of research topic since the middle of the 1990's, and many publications have been devoted to it until today, see for example the references cited in the books [15–17]. Other than the C-property, there are two other categories of steady states. The first one, studied by hydraulic engineers [18] because it is decisive for river flows, but less studied by numerical engineers because it is the equilibrium between two constant source terms, a priori relatively easy to satisfy at the discrete level, corresponds to an equilibrium. The second category of stationary states results from an equilibrium, in a linearized version of the system, between pressure term and Coriolis term. This equilibrium is known as geostrophic equilibrium. At large scales, atmospheric and oceanic flows are mostly perturbations of this