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Broad Learning System with Preprocessing to Recover the Scattering Obstacles with Far–Field Data

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Abstract. Based on Broad Learning System with preprocessing, the impenetrable obstacles were reconstructed. Firstly, the far-field data were preprocessed by Random Forest, and the shapes of the obstacles were classified by dividing the far-field data into different categories. Secondly, the broad learning system was employed for reconstructing the unknown scatterer. The far-field data of the scatterer were regarded as the input nodes of mapped features in the network, and all the mapped features were connected with the enhancement nodes of random weights to the output layer. Subsequently, the coefficient of the output can be obtained by the pseudoinverse. This method for the recovery of the scattering obstacles is named RF-BLS. Finally, numerical experiments revealed that the proposed method is effective, and that the training speed was significantly improved, compared with the deep learning method.

AMS subject classifications: 35R30, 65N21

Key words: Inverse scattering problem, broad learning system, machine learning, random forest.

1 Introduction

As one of the most important fields in applied mathematics in recent years, inverse scattering problem serves as a prototype model for various applications, such as medical imaging, ultrasonic tomography, non-destructive testing, radar, remote sensing and seismic prospecting. The uniqueness is proved in many different inverse scattering problems [3,11]. To solve the inverse scattering problem [1,2,4,10,37] by the measurement of the associated acoustic far-field data, probe [15] and linear sampling method [14,17,18] have been proposed to reconstruct the shape of obstacles. Wang et al. [36] constructed an indicator function to express the characteristics of scatterers, and then established the basic equation to solve the inverse problem, so as to determine the boundary shape of scatterers. Kirsch et al. [16] solve the problem of shape recognition in the background

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of inverse scattering problem and impedance tomography problem by a factorization method. On the basis of traditional finite element method, adaptive technology [27, 39] is added, which can be used for reconstruction of shapes. Aiming at the complex problem of reconstruction in medium, Liu et al. [24, 25] proposed some method to reconstruct the shape of obstacles. Some research results were given for the scattering problems by an exterior inhomogeneous or homogeneous medium containing an obstacle [7, 12]. Yin [44] recovered multiple obstacles and solved the interior inverse scattering problem for a two-layered cavity using the Bayesian method. In [20–22], a single-shot method was developed for various inverse scattering problems. More recently, in [9, 23], an inverse scattering reconstruction scheme based on interior resonant modes were developed. In addition, the inverse source problem [31, 38] has also been concerned in rencent years.

The acoustic inverse scattering problem is a typical nonlinear ill-posed problem. In recent years, many studies have shown that the machine learning [19, 42, 43, 45] can approach any non-linear relationship between model input and output in practical applications due to its self-learning ability. Therefore, more and more scholars pay attention to the machine learning in dealing with inverse problems [13, 33]. Shahnas et al. [32] predicted mantle flow processes in the geomechanical inverse problem by using machine learning techniques. Meng et al. [28] updated the model parameters by the self-learning method and reconstructed the obstacles based on the idea of neural network and gating. Trifonov et al. [35] solved an IP of determination of concentrations of components in multi-component solutions by their Raman spectra. And the results demonstrated by various machine learning methods are compared by the solution error and by their resilience to various types of noise encountered in experimental spectroscopy. Pilozzi et al. [29] presented a machine learning method to solve the inverse problem that may help finding optimized solutions to engineer the topology for each specific application.

The machine learning suffers from the large amount of calculation and high computational complexity. Chen et al. [8] proposed a broad learning system. The idea of the broad learning system is to use the random vector function to link neural network. This system avoids deep structure and is an efficient incremental learning. Readers are referred to [26, 30, 46] for the recent developments. This system can also be used to solve the fractional equation [5, 6, 41]. To our knowledge, there is no result by using the broad learning system to solve the inverse scattering problems. Therefore, we try to apply the broad learning system to the reconstruction of the shape of impenetrable obstacle in the present paper.

The rest of this paper is organized as follows. In Section 2, we introduce a type of scattering problems about the reconstruction of impenetrable obstacles with sound-soft boundary condition. Section 3 is devoted to our new reconstruction method by the random forest and broad learning system. In Section 4, we present some numerical experiments to verify the efficiency of the proposed method and compare with the machine learning.