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Electroelastic Analysis of Two-Dimensional Piezoelectric Structures by the Localized Method of Fundamental Solutions

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Abstract. Accurate and efficient analysis of the coupled electroelastic behavior of piezoelectric structures is a challenging task in the community of computational mechanics. During the past few decades, the method of fundamental solutions (MFS) has emerged as a popular and well-established meshless boundary collocation method for the numerical solution of many engineering applications. The classical MFS formulation, however, leads to dense and non-symmetric coefficient matrices which will be computationally expensive for large-scale engineering simulations. In this paper, a localized version of the MFS (LMFS) is devised for electroelastic analysis of two-dimensional (2D) piezoelectric structures. In the LMFS, the entire computational domain is divided into a set of overlapping small sub-domains where the MFS-based approximation and the moving least square (MLS) technique are employed. Different to the classical MFS, the LMFS will produce banded and sparse coefficient matrices which makes the method very attractive for large-scale simulations. Preliminary numerical experiments illustrate that the present LMFM is very promising for coupled electroelastic analysis of piezoelectric materials.

AMS subject classifications: 62P30, 65M32, 65K05

Key words: Localized method of fundamental solutions, meshless methods, piezoelectric structures, coupled electroelastic analysis, fundamental solutions.

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1 Introduction

Advanced components and structures that containing piezoelectric materials have been widely utilized and designed nowadays in modern engineering applications, especially in the fields of microelectromechanical systems and smart materials [1–3]. Accurate and efficient analysis of the electroelastic behavior of piezoelectric materials is a challenging task in the community of computational mechanics [4–7]. The research bottleneck arises from the basic characteristic of the piezoelectric structures: the highly electroelastic coupling of the mechanical deformation and the electric effect. The powerful and widely applied finite element method (FEM) offers without doubt many advantages in solving such problems due to its flexibilities in dealing with the complex loading type and geometry. The FEM itself, however, has also many inherent shortcomings especially when a remeshing procedure is required or when the FEM elements become highly distorted [6, 8–18]. This led to the rapid development of various meshless/ meshfree methods [7,10,19–33]. Among these methods, the method of fundamental solutions (MFS) has emerged as a popular and robust meshless boundary collocation method for the numerical solution of many engineering applications [34–44]. During the past decade, the MFS has been essentially improved and can now be considered as a competitive alternative to the FEM, especially for problems where the boundary is of prime interest, such as the free boundary problems, inverse problems and moving boundary value problems.

The classical MFS formulation, however, will produce dense and non-symmetric coefficient matrices which will be computationally expensive for large-scale engineering simulations. This is why the MFS-based methods have been limited only to solving problems with a few thousands of unknowns for a long time. In recent years, a localized version of the MFS (LMFS) has been proposed to overcome the aforementioned bottleneck of the classical MFS. The method was firstly proposed by Fan et al. [45] in 2019 and was later essentially improved and extended by many other others [11, 18, 23, 32, 46–52]. Different to the classical MFS, in the LMFS model the computational domain should be divided into a set of overlapping small sub-domains. In each of the sub-domain, the MFS-based approximation and the moving least square (MLS) technique are applied to form the local system of linear equations. The LMFS will finally produce a banded and sparse matrix system which can be solved very efficiently by using various sparse matrix solvers.

The objective of this paper is to document the first attempt to apply the LMFS for coupled electroelastic analysis of 2D piezoelectric structures. Based on the theory of piezoelectricity, the corresponding LMFS formulation are derived and the detailed information of the coupled mechanical and electrical equations are also provided. Numerical results of four benchmark numerical examples are compared with the analytical solutions and good agreement is obtained.

2 Problem statement of piezoelectricity problems