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A Non-Singular Boundary Element Method for Interactions between Acoustical Field Sources and Structures

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Abstract. Localized point sources (monopoles) in an acoustical domain are implemented to a three dimensional non-singular Helmholtz boundary element method in the frequency domain. It allows for the straightforward use of higher order surface elements on the boundaries of the problem. It will been shown that the effect of the monopole sources ends up on the right hand side of the resulting matrix system. Some carefully selected examples are studied, such as point sources near and within a concentric spherical core-shell scatterer (with theoretical verification), near a curved focusing surface and near a multi-scale and multi-domain acoustic lens.

AMS subject classifications: 65N38, 76M15

Key words: Acoustic monopoles, acoustic lens, boundary integral method, quadratic elements.

1 Introduction

Sound waves are of importance for numerous applications, ranging from speech to annoying or even hazardous noise prevention. These phenomena can be effectively modelled via the Helmholtz equation. In the frequency domain, assuming a $e^{-\mathrm{i}\omega t}$ time dependency for all quantities, with ω the angular frequency and t time, sound wave phenomena can be described by the Helmholtz equation:

$$\nabla^2\phi+k^2\phi=0, \tag{1.1}$$

where ϕ represents the (velocity) potential and k the wavenumber. The same equation is also valid for the pressure. Eq. (1.1) can be efficiently solved using a boundary element method (BEM). Using BEM for acoustics is still an active area of research [17,23,35].

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Often, the effect of the presence of one or a set of (localized) volume sources is important. For example for simulating traffic noise, airplane noise [19] or other localized sources of sound [22]. In those instances, there is no need to model an acoustic source exactly, but it can be simplified as a point source, here represented by an acoustic monopole. To implement these monopoles into numerical methods, due care has to be taken to deal with the physical singularities associated with those point monopoles.

In this work, we introduce a non-singular boundary element method for acoustics including monopoles in which the mathematically "artificial" singularities in the integrands originating from the Green's function are fully removed before any numerical procedures or calculations. In the conventional ways, the mathematical singularities from the Green's function are treated numerically by a local change of variables in the evaluation of the surface integrals [34], which accompanies with additional coding efforts. Some attempts to remove the singularities before the numerical evaluations of surface integrals introduced new unknowns for the tangential derivatives on the surface which leads to unnecessary computations [18]. Another idea to deal with the singularities needs to add additional artificial parameters on a dummy "nearby" boundary which accuracy strongly depends on the choice of that dummy boundary [36]. Other attempts to treat the mathematical singularities from the Green's function in boundary element methods include density interpolation methods [5, 21] and plane-wave singularity subtraction techniques [20].

In our non-singular boundary element method (BEM), we show that the physical point monopoles can easily be incorporated in the boundary element method formulations which ends up on the right hand side of the resulting matrix system. Such an analytical treatment is more advantageous than domain methods (volume methods) which need very careful mesh strategies to represent the area near the point source [7]. Also, boundary element methods are particularly suited for problems in infinite domains: the radiation conditions at infinity are automatically satisfied and no mesh is needed anywhere else than on the boundaries. Moreover, the non-singular boundary element method presented here takes the implementation a step forward to fully remove the mathematical singular behaviours from the Green's function before any numerical procedures without introducing any unnecessary new unknowns or dummy "nearby" boundaries. As such, the regularized integrands allow the easy implementation of high order surface elements to improve the calculation efficiency and accuracy [31]. Meanwhile, the non-singular BEM introduced here has great potential to deal with the near singularity issues or the boundary layer effect which is a main issue for the numerical process to evaluate strong or weak singularities of surface integrals by a local change of variables [26, 28, 30, 31].

The outline of the paper is as follows: in Section 2 the mathematical implementation of the inclusion of a monopole into a fully desingularized boundary element framework is described. The constructed numerical framework is then validated in Section 3 for a multi-domain concentric core shell configuration with an analytical solution given in the Appendix. From a physical point of view, interesting phenomena can arise with