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A Combination of High-Order Compact Finite Difference Schemes and a Splitting Method that Preserves Accuracy for the Multi-Dimensional Burgers' Equation

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Abstract. A class of high-order compact finite difference schemes combined with a splitting method that preserves accuracy are presented for numerical solutions of the multi-dimensional Burgers' equation. Firstly, the implicit high-order compact difference scheme is used to discretize Burgers' equation by non-linear weights that are required to be calculated at each time stage. Secondly, the sixth-order compact difference scheme in space and the fourth-order Runge-Kutta in time are applied to solve the 1D Burgers' equation. Meanwhile a linear stability analysis indicates the scheme is conditionally stable. Thirdly, the 2D and 3D Burgers' equations are divided into 1D subsystems by the splitting method, then these sub-equations' spatial terms are discretized by the fourth-order compact difference scheme, whereas the time discretizations are unchanged. The analysises of stability and accuracy of the splitting method are given to prove the accuracy of splitting without a significant loss. Finally, the accuracy and reliability of the proposed method are tested by comparing our experimental results with others selected from the available literature. It is shown that the new method has high-resolution properties and can effectively calculate Burgers' equation at large Reynolds number.

AMS subject classifications: 65M10, 78A48

Key words: Multi-dimensional Burgers' equation, high-order compact difference scheme, splitting method.

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1 Introduction

The Burgers' equation is the most fundamental nonlinear model in fluid dynamics. It is regarded as a simplified form of the Navier-Stokes equation that contain nonlinear advection and diffusion terms, which arise in computational fluid dynamics as the conceptual understanding of many physical flow phenomena such as turbulence, boundary layer behaviour, shock wave formation, etc. [1–3]. The type of equation may converge slowly for large Reynolds number, because the analytic solutions usually involve infinite series with some specific initial and boundary conditions. In general, numerical method is required to achieve accurate results for the Burgers' equation, and will in turn lead to improve fluid dynamics methods.

A variety of numerical methods for the Burgers' equation have been proposed up to now. Such as algorithms based on the modified local Crank-Nicolson method [4], spline function method [5–8], least squares reconstruction method [9], Hopf-Cole transformation and a reproducing kernel function method [10], Sonic shocks governed method [11], finite volume method [12–14], lattice Boltzmann method [15], finite volume weighted method [16,17], finite compact difference method [18,19], finite difference method [20,21], Galerkin finite element method [22,23].

In general, there are many factors that evaluate the performance of a numerical algorithm, but the most important evaluation indicators are the computational accuracy of the algorithm. In many cases, we need to solve the Burgers' equation to overcome low accuracy and low resolution. Thus, it is required to construct a numerical algorithm, which can improve calculation accuracy and avoid non-physical numerical oscillation effectively, and it needs to have high resolution. To attain this goal, high-order numerical algorithms and implicit compact difference schemes can be applied to obtain high accurate numerical solution, which has been gaining attention recently for solving partial differential equation (PDE). For example, the compact difference schemes [24–26] are proposed for solving the one-dimensional (1D) Burgers' equation, and there are some finite difference schemes [27–29] were used to solve the two-dimensional (2D) Burger's equation. Meanwhile, the three-dimensional (3D) Burgers'equation is solved by the other difference schemes [3,30,31]. These schemes have great performance in situation of smooth solution, and they possess a faster rate of convergence than the corresponding explicit schemes. In general, the numerical algorithms have poor ability to compute the case of large Reynolds number. Therefore, an important problem is how to build a scheme which approximate the solution not only in smooth regions but also in case of large Reynolds number or in the vicinity of shocks well.

In this paper, we solve the multi-dimensional Burgers' equation by an implicit high-order compact difference scheme [18] combined with a splitting method [32] that preserves accuracy and the classical fourth-order four-stage Runge-Kutta (RK4) method [19], in which, we deal with the nonlinear terms of the Burgers' equation by nonlinear weighting, so as to ensure the high accuracy and feasibility of the combined method, it carry out a process by updating nonlinear weighting of the Burgers' equation at each time stage.