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On the Equilibria of the Planar Equilateral Restricted Four-Body Problem with Radiation Pressure

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Abstract. The purpose of this work is to shed some light on the dynamics of the equilateral restricted four-body problem with equally massed radiating bodies. The positions along with the linear stability of the coplanar equilibria are determined by using numerical methods. Specifically, we conduct a rigorous and systematic analysis for elucidating the influence of the mass parameter m_3 and the radiation pressure factor q on the dynamics of the system. Our results indicate that these two parameters are very influential in the equilibria of the system.

AMS subject classifications: 65P20, 37N05

Key words: Restricted four-body problem, equilibrium points, basins of convergence.

1 Introduction

The problem describing the motion of N bodies is, without any doubt, one of the most fascinating topics of celestial mechanics and dynamical astronomy. It all started with the classic problem of three bodies [44], which introduced a new, intriguing field of research that stills remains very active.

Similarly, the problem of four bodies allow us to model several celestial systems, such as the Sun-Earth-Moon system (see e.g., [12, 14, 17, 37]), Sun-Jupiter-Saturn system (see e.g., [19, 29]), the system of Sun-Jupiter-Trojan Asteroid or Spacecraft (see e.g., [7, 9]), the

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epsilon Aurigae system (see e.g., [15, 45]), a system of a brown dwarf star, a gas giant planet and a massless Trojan (see e.g., [32]), as well as a system of a parent star, two massive planets, and a massless Trojan (see e.g., [33]).

Applications of the problem of four bodies exist also in the field of galactic dynamics. This is true if we take into account that for example in our galaxy, more than 2/3 of the observed stars form multi-star systems [30]. More precisely, triple stellar systems contain about 1/5 of the stars, while another 1/5 corresponds to higher-order systems, such as quadruple stellar systems. On this basis, the four-body problem can serve as an excellent tool for describing the dynamical properties of stars in these multi-star systems (see e.g., [18]).

A plethora of previous works is devoted to the dynamical properties of the restricted four-body problem (see e.g., [10, 48]), while more specialized studies also exist, on the equilibria (see e.g., [6, 13, 16, 20, 27, 36, 38]), as well as on families of periodic orbits (see e.g., [5,7,8,25,26,34,39]). Moreover, it should be noted, that over the years the dynamical model has been revised and plenty of modifications have been proposed for improving its applications and making it more realistic (see e.g., [1–4, 21, 22, 40–43]).

The properties (location and linear stability) of the equilibrium points in the case of two equally massed bodies have been presented in [49], while recently in [50] we expanded the analysis by considering and exploring the case of three bodies with unequal masses. The present paper follows similar logic and numerical techniques in an attempt to determine the dynamical properties of the libration points of the system, in the case where there are two equally massed bodies which are also sources of radiation.

For all the numerical calculations we shall use high-precision routines, written in standard FORTRAN 77 [28], while all the graphics of the article will be designed in the latest version 12.1 of Mathematica® software [46].

The layout of the article is the following: the theoretical description of the system dynamics is given in Section 2, while all the numerical analysis of the properties of the equilibria is presented in Section 3. Section 4 closes the paper, by emphasizing the main findings of our study.

2 The properties of the system of four bodies

The system is composed of three bodies, P_1, P_2, and P_3 which are placed at the vertices of an equilateral triangle, while they move on circular orbits, having a common and constant angular velocity ω. A fourth body P acts as a test particle and move inside the combined gravitational field of the three main bodies (see Fig. 1). The masses of the bodies are m_1, m_2, and m_3, while the mass of the fourth body m is considered significantly smaller, so as not to disturb the motion of the main bodies.

The sum of the masses of the three main bodies, along with their mutual distance R allow us to define a system of units, suitable for the specific case. In particular, we assume that $m_1+m_2+m_3=R=1$, while $G=\omega=1$.