Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 2, pp. 333-354

DOI: 10.4208/aamm.OA-2019-0370
April 2021

Temperature Effect on the Fundamental Breakdown Mechanism of Mack Mode Disturbances in Hypersonic Boundary Layers

Jiakuan Xu[1,2] and Jianxin Liu[3,*]

[1] *School of Aeronautics, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China*
[2] *Department of Mathematics, Imperial College London, SW7 2AZ, London, UK*
[3] *Department of Mechanics, Tianjin University, Tianjin 300072, China*

Received 22 December 2019; Accepted (in revised version) 23 July 2020

Abstract. In hypersonic boundary layers, Mack mode is the most unstable mode and its secondary instability is a hot research topic on the laminar-turbulent transition. Understanding the mechanism of a secondary instability is very important to delay/promote turbulence generation. In this paper, we focus on the main routes of secondary instability to turbulence in hypersonic flows, including fundamental breakdown and subharmonic breakdown, especially the former one. Through the linear and non-linear stability analyse and secondary instability analysis at various flow temperature conditions, we are trying to find out the temperature effect on the secondary instability mechanism of Mack mode disturbances. The results point out that the fundamental mode always dominates the breakdown type when the saturated amplitude of the primary Mack mode is large enough. As the stagnation temperature increases, the maximum growth-rates of the fundamental mode and subharmonic mode both increase. Meanwhile, when the wall is cooling, the maximum growth-rates of the fundamental mode and subharmonic mode are both enlarged. In contrast, with the heating wall, the maximum growth-rates of the secondary instability both decrease.

AMS subject classifications: 76E19, 76N20

Key words: Hypersonic boundary layer, secondary instability, Mack mode, fundamental mode, subharmonic mode.

1 Introduction

In two-dimensional (2D) supersonic boundary layers, first modes or Mack modes always dominate boundary layer transition. In a $\mathrm{Ma} > 4$ general hypersonic boundary layer

*Corresponding author.
Emails: shookware@tju.edu.cn (J. X. Liu), jk.xu@nwpu.edu.cn (J. K. Xu)

with an isothermal wall, the Mack mode becomes more important and probably leads to transition. The least stable Mack mode disturbance is always 2D. And it has been known that the secondary instability plays an important role in the transition process from 2D disturbances to three-dimensional(3D) turbulence. In order to study this problem, Herbert [1] proposed a secondary instability theory based on the Floquet expansion. According to different Floquet parameters, the unstable modes in a secondary instability can be classified into three types that include the fundamental mode, subharmonic mode and detuned mode [2]. The fundamental mode, often leads to the fundamental breakdown, having the same streamwise wavelength and frequency as the primary 2D disturbance. Different from the fundamental one, the subharmonic mode has twice the streamwise wavelength and one half the frequency as the primary mode disturbance. It can cause the subharmonic breakdown, i.e., H/N-type breakdown. In the subharmonic breakdown, the turbulent spots are always staggered. Detuned modes are the modes between the fundamental and subharmonic ones, which can be observed in the literature [3]. In the past several decades, researchers have paid much attention to the theoretical and experimental studies on the typical subharmonic breakdown and fundamental breakdown.

On one hand, about the investigation of the subharmonic breakdown (H/N type), triad resonance theory is firstly proposed by Craik [4] in incompressible flows. Then, Herbert's secondary instability theory based on Floquet theory mentioned above gives a reasonable explanation for the subharmonic resonance [1]. In 1977, Kachanov et al. [5] firstly observed the N-type subharmonic breakdown in the experiment. All these studies point out that a subharmonic mode usually dominates the secondary instability in incompressible flows. Nevertheless, in high-speed flows, there are more complex routes of breakdown to turbulence, and the subharmonic mode also plays a very important role in these flows. Kosinov and his co-workers [6,7] conducted lots of measurements on the subharmonic resonance mechanism in supersonic boundary layers. In 1992, Ng and Erlebacher [8], Masad and Nayfeh [9] and El-Hady [10] investigated the secondary instability in compressible boundary layers by the Floquet theory and pointed out that at a high Mach number of 4.5, the secondary instability of a primary Mack mode is stronger than that of a primary first mode, and the subharmonic resonance dominates the secondary instability.

On the other hand, the so-called K type breakdown was investigated in moderate and high turbulence density freestream conditions in the incompressible experiment [11]. The fundamental breakdown, which is caused by the fundamental mode, is also observed in hypersonic flows. From 2009, Schneider's group has conducted a series of experiments [12] on a flared cone in the Boeing/AFOSR Mach 6 Quiet Tunnel (BAM6QT) at Purdue University. The measurements indicate that the streaky structures can be found in the earlier stage of the transition and the azimuthal wave number is in consistent with the fundamental mode. This characteristic suggests that the fundamental mode, not the subharmonic one, may dominate the secondary instability on the flared cone in a hypersonic flow. A long term experimental study has also been performed and reveals a