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Closed Form Solutions of the Perturbed Gerdjikov-Ivanov Equation With Variable Coefficients

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Abstract. The solutions of the perturbed Gerdjikov-Ivanov equation with coefficients depending on temporal variable are obtained. They are presented in terms of generalised solitary and periodic solutions and can be used to determine new solutions of the classical perturbed Gerdjikov-Ivanov equation with special sets of constant coefficients.

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1. Introduction

It is well known that certain solutions of nonlinear partial differential equations provide a better understanding of various phenomena. From the mathematical point of view, the use of generalised models is important in the sense that in particular cases the solutions are derived by changing model parameters. In this work, we use the improved tanh-coth method [20] in order to find exact traveling wave solutions of the following generalised perturbed Gerdjikov-Ivanov equation

$$\begin{aligned} &iq_t + A(t)q_{xx} + B(t)|q|^4q + i \\ &\quad \times \Big[C(t)q^2q_x^* + \rho(t)q_x + \delta(t)\big(|q|^{2m}q\big)_x + \mu(t)\big(|q|^{2m}\big)_x q\Big] = 0, \end{aligned} \tag{1.1}$$

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where $q = q(x,t)$ is a complex-valued wave profile depending on spatial and temporal variables x and t, and q^* denotes the complex conjugate of q — cf. [9,14,15]. If the coefficients A, B, C, ρ, δ and μ are constants, then (1.1) is the standard perturbed Gerdjikov-Ivanov equation — cf. [6,21,26]. In this case, A is the group velocity dispersion coefficient, B the quintic nonlinearity coefficient, C the nonlinear dispersion coefficient, ρ the inter-modal dispersion coefficient, δ the self-steepening coefficient, μ the higher-order dispersion coefficient and $\imath := \sqrt{-1}$. It is worth noting that for non-constant coefficients, the structure of the corresponding solutions of (1.1) differs from that for the standard Gerdjikov-Ivanov equation. For more details the reader is referred to [3, 10–13, 16, 27]. We note that the easily numerically implementable improved tanh-coth method of [20] can be considered as a generalisation of classical methods such as the tanh-coth method [24], the Kudryashov method [8], the G'/G method [23], the $\mathrm{Exp}(-\phi(\xi))$ method [4] and the methods considered in [1,7,17–19,22,25].

This paper is organised as follows. In Section 2 we review the improved tanh-coth and Exp-function methods for nonlinear partial differential equations. New exact traveling wave solutions of the Eq. (1.1) are derived in Section 3. Finally, some conclusions are given in Section 4.

2. Description of the Methods

Given a nonlinear partial differential equation

$$P(u, u_x, u_t, u_{xt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is the unknown function, x the spatial variable, t the temporal variable, and the coefficients of (2.1) depend on the variable t only, transformations similar to the transformation

$$\xi = x + \lambda t + \xi_0$$

reduce the Eq. 2.1 to an ordinary differential equation

$$P_1(u, u', u'', \ldots) = 0 \tag{2.2}$$

with an unknown function $u = u(\xi)$.

2.1. An improved tanh-coth method

We consider a tanh-coth method consisting in finding the solutions of (2.2), which can be represented in the form

$$u(\xi) = \sum_{i=0}^{M} a_i(t)\phi(\xi)^i + \sum_{i=M+1}^{2M} a_i(t)\phi(\xi)^{M-i}, \tag{2.3}$$

where M is a positive integer to be determined later and $\phi = \phi(\xi)$ satisfies the general Riccati equation

$$\phi'(\xi) = \gamma(t)\phi^2(\xi) + \beta(t)\phi(\xi) + \alpha(t). \tag{2.4}$$