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Dependence Analysis of the Solutions on the Parameters of Fractional Delay Differential Equations

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Abstract. In this paper, we investigate the dependence of the solutions on the parameters (order, initial function, right-hand function) of fractional delay differential equations (FDDEs) with the Caputo fractional derivative. Some results including an estimate of the solutions of FDDEs are given respectively. Theoretical results are verified by some numerical examples.

AMS subject classifications: 34A08, 34A10, 34A12, 34K37
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1 Introduction

In the recent past years, the use of differential equations of fractional order (FDEs) has gained considerable popularity in several areas such as nonlinear oscillation of earthquake (cf. [1]), fluid-dynamic traffic model (cf. [2]), material viscoelastic theory and physics (cf. [3–6]), etc. In [8,9], some results about the dependence of the solutions on the parameters (including the order of the differential equation, the initial function and the right-hand function) of some classes of fractional differential equations (FDEs) with Riemann-Liouville (R-L) fractional derivatives were given.

In this paper, our aim is to extend some results about the dependence in [9] and to consider the dependence of the solutions on the above parameters of the following initial value problem of a FDDE in the form

$$\begin{cases} {}_0^C D_t^\alpha x(t) = f(t, x_t), & t \in [0, T], \\ x(t) = \varphi(t), & t \in [-\tau, 0], \end{cases} \tag{1.1}$$

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where $0 < \alpha < 1$, $\tau > 0$, $f : D = [0, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $f(t, x_t) = f(t, x(t), x(t-\tau))$, and ${}_0^C D_t^\alpha$ denotes the Caputo fractional derivative of order α and is defined in [6] as

$$ {}_0^C D_t^\alpha y(t) = I^{1-\alpha} \frac{dy(t)}{dt} = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} \frac{dy(\tau)}{d\tau} d\tau, \quad t > 0, \tag{1.2} $$

where I^α denotes the integral operator of order α and is defined in [6] as

$$ I^\alpha y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} y(\tau) d\tau, \quad t > 0, \quad \alpha > 0. $$

As we all know, there are some different definitions of fractional operator except the Caputo fractional derivative. From a theoretical point of view the most natural approach is the Riemann-Liouville definition defined in [6] as

$$ {}_0^R D_t^\alpha y(t) = \frac{d}{dt}(I^{1-\alpha} y(t)) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} y(\tau) d\tau, \quad t > 0. \tag{1.3} $$

The relationship between the Caputo definition and the Riemann-Liouville definition can be given by the following formula (cf. [5])

$$ {}_0^R D_t^\alpha y(t) = {}_0^C D_t^\alpha y(t) + \frac{y(0)}{\Gamma(1-\alpha)} t^{-\alpha}, \quad t > 0. \tag{1.4} $$

Thus, the Problem (1.1) can be written as

$$ \begin{cases} {}_0^R D_t^\alpha (x(t) - x(0)) = f(t, x_t), & t \in [0, T], \\ x(t) = \varphi(t), & t \in [-\tau, 0]. \end{cases} \tag{1.5} $$

As in [7], the Problem (1.5) have the following form

$$ \begin{cases} x(t) = \varphi(0) + \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^t (t-s)^{\alpha-1} f(s, x_s) ds, & t \in [0, T], \\ x(t) = \varphi(t), & t \in [-\tau, 0]. \end{cases} \tag{1.6} $$

In [7], existence and uniqueness of solutions to Problems (1.1), (1.5), (1.6) were given. In this paper, we assume that the existence and uniqueness of solution of Problem (1.1) hold.

This paper is organized as follows. In Section 2, some results about dependence of the solutions on the parameters of FDDEs are given, and we also give the estimate of the solutions of FDDEs. In Section 3, we identify our some theoretical results by some examples.

2 The main results

In this section, we shall present and prove our main results. Firstly, we introduce the following Lemmas and define the norm

$$ \|u(t)\|_\infty = \max_{0 \le t \le T} |u(t)|, \quad \forall u(t) \in C[0, T]. $$