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A Boundary Meshless Method for Solving Heat Transfer Problems Using the Fourier Transform

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Abstract. Fourier transform is applied to remove the time-dependent variable in the diffusion equation. Under non-harmonic initial conditions this gives rise to a non-homogeneous Helmholtz equation, which is solved by the method of fundamental solutions and the method of particular solutions. The particular solution of Helmholtz equation is available as shown in [4, 15]. The approximate solution in frequency domain is then inverted numerically using the inverse Fourier transform algorithm. Complex frequencies are used in order to avoid aliasing phenomena and to allow the computation of the static response. Two numerical examples are given to illustrate the effectiveness of the proposed approach for solving 2-D diffusion equations.

AMS subject classifications: 65Y04, 35K05

Key words: Transient heat transfer, meshless methods, method of particular solutions, method of fundamental solutions, frequency domain, Fourier transform.

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1 Introduction

Over the past four decades researchers have proposed a variety of numerical techniques to solve heat transfer problems. The Finite Element Method (FEM) and the Finite Difference Method (FDM) are well-established techniques that have often been implemented to solve these types of problems. However, they require the discretization of the full domain, which leads to problems that can be tedious and computationally costly, particularly for unbounded or high dimensional irregular domains. Different numerical techniques, such as the Boundary Element Method (BEM), have been

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developed to alleviate these computational difficulties by reducing the discretization of the problem domain to the material interfaces. However, the BEM requires the prior knowledge of fundamental solutions, and it leads to integrations along boundary elements that may be singular or even hyper-singular. Furthermore, the discretization of a three-dimensional surface is still not an easy task. More recently, attention has been focused on the development of meshless methods which require neither domain nor boundary discretization. Among these methods, the method of fundamental solutions (MFS) has emerged as an effective boundary-only meshless method for solving homogeneous equations [7,9,11]. Coupled with radial basis functions (RBFs), the MFS can be extended to solve nonhomogeneous equations, nonlinear equations, and time-dependent problems [5–7,10–12].

Most of the techniques that have been implemented to solve transient heat transfer problems use time marching schemes [1,3,14,17,20], or Laplace transforms [5,18,19,21]. The Laplace transform technique replaces the time dependence by a transform variable. However, the numerical inverse Laplace transform is ill-posed, which means that small truncation errors are magnified in the numerical inversion process. Despite the progress in numerical inversion techniques of the Laplace transfrom in recent years, the difficulty remains. The purpose of this paper is to apply the Fourier transform to remove the time dependent variable. As a result, the given heat transfer problem is reduced to a nonhomogeneous Helmholtz equation, which can be solved using the boundary meshless method mentioned above. The method of particular solution is implemented to solve nonhomogeneous Helmholtz equation in the frequency domain. In this process, the derivation of close form particular solution is crucial and is not an easy task. A particular solution for Helmholtz-type equations was originally proposed by Chen and Rashed [4] using thin plate splines and later generalized to polyharmonic splines by Muleshkov et al. [15]. The homogeneous solution is obtained by the standard MFS. Finally, time solutions are obtained by applying an inverse Fourier transform algorithm. To avoid aliasing phenomena, complex frequencies are introduced into the problem which also allows the computation of the static response. The effect of the presence of these complex frequencies is removed in the time domain by using an exponential window to rescale the response.

In Section 2 we first define the two-dimensional heat transfer problems and describe how to convert it to Helmholtz equation using Fourier transform. In Section 3, we briefly explain how to obtain the particular solution using thin plate splines. In Section 4, the MFS for solving the homogeneous solutions is described. In Section 5, we give a brief account of how the time solutions are obtained using inverse Fourier transform. In Section 6, the proposed technique is verified by solving two different heat transfer problems in a rectangular domain for which analytical solutions are known. We first consider constant initial conditions in the full domain and the verification procedure is performed in the time domain, while the second problem assumes a non-constant temperature distribution in the full inner domain. For this latter case, other than time domain comparisons against explicit results, verifications of the responses in the frequency domain are also presented.