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Valuation of Basket Credit Default Swaps Under Stochastic Default Intensity Models

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Abstract. Portfolio credit derivatives, including the basket credit default swaps, are designed to facilitate the transfer of credit risk amongst market participants. Investors consider them as cheap tools to hedge a portfolio of credits, instead of individual hedging of the credits. The prime aim of this work is to model the hazard rate process using stochastic default intensity models, as well as extend the results to the pricing of basket default swaps. We focused on the nth-to-default swaps whereby the spreads are dependent on the nth default time, and we estimated the joint survival probability distribution functions of the intensity models under the risk-neutral pricing measure, for both the homogeneous and the heterogeneous portfolio. This work further employed the Monte-Carlo method, under the one-factor Gaussian copula model to numerically approximate the distribution function of the default time, and thus, the numerical experiments for pricing the nth default swaps were made viable under the two portfolio types. Finally, we compared the effects of different swap parameters to various nth-to-default swaps.

AMS subject classifications: 91G20, 91G30, 91G40, 91G60, 91G70, 62P05, 65C05

Key words: Portfolio credit derivatives, basket default swaps, Gaussian copula, Monte-Carlo simulations, stochastic intensity modelling, hazard rate, joint survival probability distribution.

1 Introduction

Basket default swaps (BDS) are financial contracts that payoff whenever there is a default or multiple defaults among a portfolio of entities or obligors. BDS are generally classified into first-to-default, nth-to-default, n-out-of-m-to-default and all-to-default. From

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the investor's point of view, BDS are still preferable because they limit the credit portfolio that an investor can easily monitor compared to the large portfolio obtainable in a typical synthetic collateralised debt obligation (CDO)[†] [1]. The protection buyer finds the BDS attractive because the cost of purchasing protection on a portfolio of entities is less expensive compared to the purchase of individual protection. Considering the nth-to-default (n2D), the protection seller enjoys a limited downside risk, and this stems from the fact that at most one default is expected for the seller to cover [7]. After the payments pending default and contract finalized, the loss incurred resulting from further default is borne only by the protection buyer. The recovery rate, number of entities in the portfolio, the entity's credit ratings, as well as the default correlations are some of the major factors that affect the BDS spread.

Several research works have been carried out in the area of basket default swaps. David (2000) first implemented the copula functions to model credit derivatives, and this was extended by several authors. For instance, Jean-Paul and Jon (2005) considered the valuation of BDS and the CDO. They obtained semi-analytic values for the above contingent claims under the assumption of independent default times conditioned on a low dimensional factor, and based their valuation on mean-variance mixture models and frailty models. Masaaki and Yukio (2000) priced BDS using a stochastic intensity-based model under the assumption of conditional independence. Using the joint survival probability, they further provided closed form values when the intensity process is defined within the extended Vasicek model.

Jin et al. (2011) further provided the limitation of using the Vasicek model in modelling the intensity process. They explained using some numerical examples that the Vasicek model can be efficient when the portfolio has relatively few correlated risky assets and their valuations are extended to the prices of BDS, credit default swap (CDS) index and CDOs. Abid and Naifar (2007a) provided Monte-Carlo methods and semi-explicit expressions which improves the cost-effectiveness of the Monte-Carlo approach to price multi-named credit derivatives like the basket default swaps and CDO tranches. They further calibrated the prices of BDS to the Japanese financial markets [13]. Ian and Alex (2006) proposed a recursive algorithm to value BDS based on a continuous-time model in the conditional independence framework. They employed the concept of the order statistics of the default times of the entities in a portfolio, and then applied it in the estimation of first-to-default and the second-to-default contracts.

Regardless of whether the reduced form method or the structural approach in modelling the joint default events in portfolio credit derivatives, there is always the problem of the joint probability distribution of the default times, and many researchers have employed Copula models. Rüdiger and Alexander (2003) explored the role of copulas in latent variable models and used the modified Gaussian copula method to model the dependent defaults evident in a portfolio credit risk. Roy and Marco (2002) used the student t-copulas, which posses the non-trivial tail dependency structure, as well as the ability for

[†]Small scale portfolio size includes 5-10 credits, and large portfolio scale ranges from 100-150 credits.