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The Modified Localized Method of Approximated Particular Solutions for Linear and Nonlinear Convection-Diffusion-Reaction PDEs

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Abstract. In this paper, a kernel based method, the modified localized method of approximated particular solutions (MLMAPS) [16, 23] is utilized to solve unsteady-state linear and nonlinear diffusion-reaction PDEs with or without convections. The time-space and spatial space are discretized by the higher-order Houbolt method with various time step sizes and the MLMAPS, respectively. The local truncation error associated with the time discretization is $\mathcal{O}(h^4)$, where h is the largest time step size used. The spatial domain is then treated by a special kernel, the integrated polyharmonic splines kernels together with low-order polynomial basis. Typical computational algorithms require a trade off between accuracy and rate of convergency. However, the experimental analysis has shown high accuracy and fast convergence of the proposed method.

AMS subject classifications: 65N35, 65N99

Key words: Polyharmonic spline, Houbolt method, time-dependent PDEs, method of approximated particular solutions, MLMAPS, convection-diffusion-reaction, nonlinear, kernel methods.

1 Introduction

The domain type meshfree methods utilizing radial basis functions, such as Kansa's method [11, 12] and the method of approximated particular solutions (MAPS) [3, 4], are classified as global meshfree methods because the methods result in the creation of a dense linear system. Yao et al. further localized the MAPS into the localized MAPS (LMPAS) [21] which allows the creation of a sparse system. This is especially useful for

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solving large-scale problems. The LMAPS utilizes the collocation scheme on overlapping local domains with integrated radial basis functions (RBFs), such as multiquadrics, inverse multiquadrics, and Matern. This technique drastically reduces the storage size of the collocation matrix. This improved the computational efficiency of the method for solving large-scale partial differential equations (PDEs). This allows the LMAPS to compete with the traditional numerical methods such as the finite element method (FEM) for large-scale problems. Since then, the LMAPS has been applied to various kinds of differential equations, such as the biharmonic equation [15], near-singular PDEs [24], the unsteady Burgers' equations [17, 27], convection-diffusion equations [2], 3D nonlinear Schrödinger equations [18], and wave equations [10], as well as unsteady Navier-Stokes problem [7, 26].

A modified LMAPS (MLMAPS) presented in [22] shows a significant improvement in terms of accuracy by using integrated polyharmonic splines in radial space together with the polynomial basis for linear and nonlinear elliptic PDEs in 2D and 3D. Since then, the method has been applied to time-dependent PDEs [16]. On the other hand, the Houbolt method in [9, 19, 25] is a high-order accurate time discretization scheme. Note that MLMAPS can be classified as a kernel-based method, which has similar ideas as the generalized finite difference method or radial basis function- finite difference method. The main difference is that MLMAPS uses commonly used kernels in the integrated forms, in addition to low-order polynomial basis. This is a combination of basis functions that amazingly preserve the high accuracy of the polynomial basis and flexibility of the kernel based methods.

In this paper, MLMAPS is coupled with the Houbolt method to solve linear or nonlinear diffusion-reaction PDEs with or without convection terms:

$$\frac{\partial u}{\partial t}=\mathcal{D}u+f(\boldsymbol{x},t), \qquad \boldsymbol{x}\in\Omega\subset\mathbb{R}^d, \tag{1.1}$$

with the boundary condition $\mathcal{B}u(\boldsymbol{x},t)=g(\boldsymbol{x},t)$, $\boldsymbol{x}\in\partial\Omega$, and the initial condition $u(\boldsymbol{x},0)=u^0$, $\boldsymbol{x}\in\Omega\cup\partial\Omega$, where $\mathcal{D}$ is linear or nonlinear diffusion-convection differential operator, f is a reaction function, and $\mathcal{B}$ is a linear or nonlinear boundary differential operator, functions f, g and u_0 are known, with physical domain Ω in $\mathbb{R}^d$. We will combine the implicit Euler method, the Houbolt method, and MLMAPS to solve this type of PDEs.

The rest of the paper is organized as follows: In Section 2, the first few time-steps are discretized by the traditional implicit time-stepping method with small evenly-spaced time-step h_0. This allows the Houbolt method to be used in the following time-steps after first three steps, the time step will jump from h_0 to a relatively larger time-step h. After transitioning to h, the third order Houbolt method will be used for the evenly-spaced time discretization with time-step h. The error analysis associated with time discretization is presented at the end of this section. The time discretization transforms the given time-dependent PDE to a series of elliptical differential equations. Therefore, in Section 3, the spatial discretization using MLMAPS with integrated polyharmonic splines together with polynomial basis are introduced. Section 4 illustrates the performance of the nu-