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Two-Dimensional Legendre Wavelets for Solving Time-Fractional Telegraph Equation

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Abstract. In this paper, we develop an accurate and efficient Legendre wavelets method for numerical solution of the well known time-fractional telegraph equation. In the proposed method we have employed both of the operational matrices of fractional integration and differentiation to get numerical solution of the time-telegraph equation. The power of this manageable method is confirmed. Moreover the use of Legendre wavelet is found to be accurate, simple and fast.

AMS subject classifications: 65T60, 26A33

Key words: Telegraph equation, Legendre wavelets, fractional calculus, Caputo derivative.

1 Introduction

Fractional ordinary and partial differential equations, as generalizations of classical integer order differential equations, are increasingly used to model problems in fluid flow, mechanics, viscoelasticity, biology, physics, engineering and other applications (for example see [1–3]). Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes [4–9]. Fractional differentiation and integration operators are also used for extensions of the diffusion and wave operators [10]. The solutions of fractional differential equations are much involved, because in general, there exists no method that yields an exact solution for fractional differential equations, and only approximate solutions can be derived using linearization or perturbation methods.

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Wavelet methods have been applied for solving partial differential equations (PDEs) from the beginning of 1990s [11]. In the last two decades this method of solution for such problems has attracted great attention and numerous papers about this topic have been published. Due to this fact we must confine somewhat our analysis; in the following only PDEs of mathematical physics and of elastostatics are considered. From the first field of investigation [12–17] can be cited, and for elasticity problems we refer to [18–24]. In these papers different wavelet families have been applied. In most cases the wavelet coefficients have been calculated by the Galerkin or collocation method, for which we have to evaluate integrals of some combinations of the wavelet functions (also called connection coefficients).

We consider the time-fractional telegraph equation of order α $(1<\alpha\leq 2)$ as:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}+\frac{\partial^{\alpha-1}u(x,t)}{\partial t^{\alpha-1}}+u(x,t)=\frac{\partial^2 u(x,t)}{\partial x^2}+f(x,t),\quad a\leq x\leq b,\quad t\geq 0, \tag{1.1}$$

where $\partial^\beta/\partial t^\beta$ denotes Caputo fractional derivative of order β, that will be described in the next section. This equation is commonly used in the study of wave propagation of electric signals in a cable transmission line and also in wave phenomena. This equation has been also used in modeling the reaction-diffusion processes in various branches of engineering sciences and biological sciences by many researchers (see [25] and references therein).

The fractional telegraph equation has recently been considered by many authors. Cascaval et al. [26] have discussed the time-fractional telegraph equations, and have investigated its wellposedness and asymptotic behavior by using the Riemann-Liouville approach. Orsingher and Beghin [27] discussed the time-fractional telegraph equation and telegraph processes with Brownian time, showing that some processes are governed by time-fractional telegraph equations. Chen et al. [28] also discussed and derived the solution of the time-fractional telegraph equation with three kinds of nonhomogeneous boundary conditions, by the method of separations of variables. Orsingher and Zhao [29] considered the space-fractional telegraph equations, obtaining the Fourier transform of its fundamental solution and presenting a symmetric process with discontinuous trajectories, whose transition function satisfies the space-fractional telegraph equation. Momani [30] discussed exact and approximate solutions of the space- and time-fractional telegraph differential equations by means of the so-called Adomian decomposition method.

The aim of the present work is to develop Legendre wavelets method with both of the operational matrices of integration and differentiation for solving the time-fractional telegraph equation, which is fast and mathematically simple and guarantees the necessary accuracy for a relatively small number of grid points. The outline of this article is as follows: In Section 2 we describe properties of Legendre wavelets. In Section 3 the proposed method is used to approximate the solution of the problem. In Section 4 some numerical examples are solved by applying the method of this article. Finally a conclusion is drawn in Section 5.