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Variance Swap Pricing under Hybrid Jump Model

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Abstract. This paper investigates the pricing of discrete-sampled variance swaps driven by a generalised stochastic model taking into account stochastic volatility, stochastic interest rate and jump-diffusion process. The model includes various existing models as special cases, such as the CIR model, the Heston CIR model, and the multi-factor CIR model. The integral term arising from the jump-diffusion is dealt with by employing the characteristic function and Fourier convolution. By applying a high-dimensional generalised hybrid method, a semi-analytic solution is derived. The effects of stochastic interest rate, stochastic volatility and jump rate on variance swap price are investigated. It is shown that both the stochastic volatility and the jump rate have significant effects on the fair strike price, while the effect of the stochastic interest rate is minor and can be ignored.

AMS subject classifications: 91G20

Key words: Time-scale, stochastic volatility, generalised Fourier transform, variance swap.

1. Introduction

Variance and volatility swaps are well-known financial derivatives which allow investors to trade the realised volatility against the current implied volatility. With variance swap and volatility swap, investors can hedge and speculate risk from the asset price movement. On future realised price variance, a variance swap is a forward contract with the following payoff function

$$\text{Payoff} = \mathbb{L}\left(\sigma_R^2 - K\right),$$

where $\mathbb{L}$ denotes the notional amount of the swap per annualised volatility point, σ_R is the realised volatility and K is the annualised strike price [6].

A long position in a variance swap gives benefit when the realised volatility is higher than the strike price, while a short position will benefit when the realised volatility is lower

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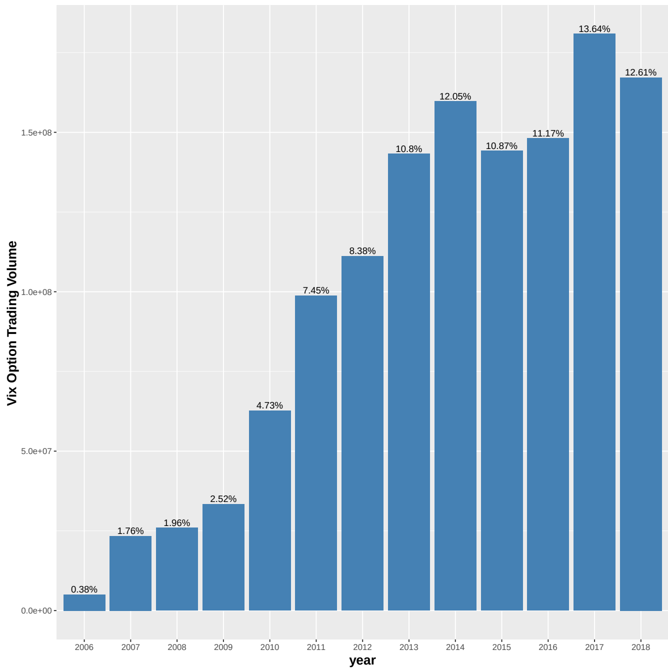

Figure 1: VIX Options Year Volume from the Chicago Board Options Exchange(CBOE) website http : //www.cboe.com/vix.

than the strike price. The first volatility swap was traded in 1998 and has flourished recently. Fig. 1 shows the historical trading volume. Demeterfi *et al.* [16] listed two main reasons to trade volatility derivatives such as the variance swap and the volatility swap. Firstly, investigators may take a long-short position in a variance swap to hedge the risk exposure of trading volatility. Secondly, the variance swap provides them with a possibility to speculate in the spread of the realised volatility and the implied volatility.

1.1. Pricing strategies of variance swap

To price the variance swap, it is very important to understand the difference between the realised volatility and the implied volatility. The implied volatility represents the market price of volatility, it can be calculated as the inverse function of the Black-Sholes pricing formula. The realised variance, denoted by σ_R^2, is calculated from the historical data of option prices via the following equation [6]

$$\sigma_R^2 = \frac{AF}{N} \sum_{i=0}^{N-1} \left(\frac{S_{i+1} - S_i}{S_i} \right)^2, \tag{1.1}$$

where S_{i+1} denotes the underlying stock price at the $(i+1)$-th observation time, and N denotes the number of observations. Let T be the life time of the contract, $AF = N/T$ is the annualised factor converting this expression to an annualised variance, which is assumed