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Pricing American Options under Regime-Switching Model with a Crank-Nicolson Fitted Finite Volume Method

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Abstract. A new numerical method for pricing American options under regime-switching model is developed. The original problem is first approximated by a set of nonlinear partial differential equations. After that a novel fitted finite volume method for the spatial discretisation of the nonlinear penalised system of partial differential equations is coupled with the Crank-Nicolson time stepping scheme. It is shown that the discretisation scheme is consistent, stable, monotone and hence convergent. In order to solve nonlinear algebraic systems, we apply an iterative algorithm and show its convergence. Numerical experiments demonstrate the convergence, efficiency and robustness of the numerical method.

AMS subject classifications: 65M06, 65M12, 65M32, 91G60

Key words: American option pricing, regime switching, fitted finite volume method, Crank-Nicolson scheme.

1. Introduction

Since its introduction, the Hamilton regime-switching model [12] attracted wide attention of mathematicians and financial engineers [4, 8, 13]. According to this model, market may switch from one regime to another, which allows to explain periodic caused by a short-term political or economic uncertainty. Thus econometric analysis shows that the blending the regime switching component and the log-normal dynamics of the stock prices better fits the asset price dynamics [7]. Moreover, regime switching models are intuitively attractive and computationally inexpensive. Therefore, they find numerous applications in

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other fields, including electric markets [3], forestry valuation [6], and the valuation of stock loans [28].

The American options pricing problem under regime-switching model (2.1) is more complicated than other options pricing problems [20,21]. It is based on a system of coupled parabolic partial differential complementarity problems (PDCPs), so that efficient numerical methods are required to solve it. We note that various numerical methods for problem (2.1) have been studied recently. In particular, Yang [25] established stability estimates for finite element and simple lattice methods. Liu [19] developed and employed an efficient tree method for the Heston's stochastic volatility model. Khaliq *et al.* [17] introduced a fast numerical scheme, which uses the penalty and exponential time differencing Crank-Nicolson (ETD-CN) methods. Yousuf *et al.* [27] considered a second-order method based on exponential time differencing approach and studied its stability and convergence. Egorova *et al.* [9] used a multivariable front-fixing transformation to develop a conditionally stable first order in time and second order in space method. Zhang *et al.* [29] introduced a fitted finite volume method for the spatial discretisation along with a fully implicit time stepping scheme for PDEs. Subsequently, Zhang and Yang [33] provided a detailed analysis on the power penalty approach to coupled parabolic PDCPs arising in the valuation of American options under regime-switching. Xing and Ma [24] studied the convergence rates of trinomial tree methods (TTMs) and perturbed finite difference methods (PFDMs) for American options under regime-switching model. This work provides a link between probability and deterministic approaches.

The fitted finite volume method was first used in price stock options governed by the standard Black-Scholes equation [22]. Later on it was extended to other types of options — cf. [5,14,30–32] and references therein. The method combines finite volume formulations with a fitted approximation technique. It overcomes the drift dominated phenomena and gained popularity in option pricing. The fitted finite volume method in [29] evaluates the American options under regime-switching model and the corresponding PDEs are treated by a fully implicit time stepping scheme. We note that the scheme has the first-order convergence rate and the numerical tests carried out in [29] consider two-regime cases only.

In this work, a numerical method for pricing American options under regime-switching model is studied. Based on a penalty approach, the PDCPs (2.1) are reduced to a set of coupled nonlinear PDEs. Afterwards, fitted finite volume method in space with the Crank-Nicolson method in time is applied to American options under regime-switching model. We show that the numerical scheme is consistent, stable and monotone. This guaranties the convergence of the numerical solution to the viscosity solution of continuous problem. In order to solve the discretised nonlinear system, we construct an iterative method and prove its convergence. A number of numerical experiments carried out for two-, three- and four-regime American options models show the accuracy and robustness of the method proposed.

The paper is organised as follows. In Section 2, we introduce American options under regime-switching model and describe the penalty approach to the problem (2.1). Section 3 deals with the Crank-Nicolson fitted finite volume scheme. The convergence of this numerical scheme is studied in Section 4. In Section 5, we construct an iterative algorithm for