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Novel Interaction Phenomena of Localised Waves in the $(2+1)$-Dimensional HSI Equation

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Abstract. Localised interaction solutions of the $(2+1)$-dimensional generalised Hirota-Satsuma-Ito equation are studied. Using the Hirota bilinear form and Maple symbolic computations, we generate three classes of lump solutions. Specific sets of parameters are chosen to show the dynamic characteristics and evolution of lump and interaction solutions and their energy distribution.

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Key words: (2+1)-dimensional generalised Hirota-Satsuma-Ito equation, lump solution, interaction solution, Hirota's bilinear method.

1. Introduction

The systematic study of non-linear phenomena started in 1960s and experienced rapid growth since then. The solitons, in particular, found various applications in physics, biology, medicine, oceanography, economics and population problems. However, the finding of exact solutions of nonlinear systems requires a lot of effort and the approaches used include the inverse scattering transformation [1], the Bäcklund transformation [15, 16], the Darboux transformation [14], the variable separation [6], the Hirota bilinear methods [4] and some others [2, 7, 8]. One of these techniques — viz. the Hirota bilinear method has been recently utilised to establish lump solutions. Thus Wazwaz [17, 18] employed it to derive multiple soliton solutions of BKP and generalised Ito equations. Zhao [20] applied the Hirota bilinear form to investigate two and three soliton solutions of a multi-component higher-order Ito equation. The lump solution is a rational function solution, which decays in all directions of space variables [9]. Such solutions of partial differential equation (PDE) describe important wave phenomena and are determined for various classes of integrable equations [3, 11, 19]. Thus Ma [10] obtained lumps and interaction solutions of linear

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partial differential equations in $(3+1)$-dimensions. A general approach to finding of positive quadratic solutions of bilinear equations is given in [13]. On the other hand, recently Zhou *et al.* [21] considered the $(2+1)$-dimensional Hirota-Satsuma shallow water wave equation.

In this work, we study the lump and interaction solutions of the $(2+1)$-dimensional generalised Hirota-Satsuma-Ito equation

$$v_t + u_{xxt} + 3(uw)_x + \gamma \cdot u_x = 0, \quad u_y = v_x, \quad u_t = w_x, \tag{1.1}$$

where u,v,w are the function of x,y,t and the subscripts denote partial derivatives with respect to scaled space coordinates x, y and time t, cf. [5, 12]. This equation arises in the shallow water wave theory and in the Jimbo-Miwa classification.

This paper is structured as follows. In Section 2, we write the bilinear form of the HSI equation through a dependant transformation and lump and interaction solutions of the HSI equation are obtained with assistance of Maple symbolic computations. A number of lump solutions are graphically shown in order to describe the dynamics and properties. A brief conclusion is provided in Section 3.

2. Lump and Interaction Solutions of the Eq. (1.1)

Using the mapping,

$$u = 2(\ln f)_{xx}, \quad v = 2(\ln f)_{xy}, \quad w = 2(\ln f)_{xt}, \tag{2.1}$$

where $f(x,y,t)$ is a real function, we transform the Eq. (1.1) into the bilinear form

$$\left(D_y D_t + D_x^3 D_t + \gamma D_x^2\right) f \cdot f = 0. \tag{2.2}$$

For any positive integers m,n and q, the operator D is defined by

$$\begin{aligned} &D_x^m D_y^n D_t^q f(x,y,t) g(x,y,t) \\ =&(\partial_x - \partial_{x'})^m (\partial_y - \partial_{y'})^n (\partial_t - \partial_{t'})^q f(x,y,t) g(x',y',t')|_{x'=x,y'=y,t'=t}, \end{aligned}$$

and g is the function of the formal variables x',y' and t'.

In order to obtain the interaction solutions of the Eq. (1.1), we choose the function f in the form

$$\begin{aligned} f(x,y,t) =& g^2 + h^2 + a_9 \cos(a_{10}x + a_{11}y + a_{12}t + a_{13}) \\ &+ a_{14}\cosh(a_{15}x + a_{16}y + a_{17}t + a_{18}) \\ &+ \exp(-(a_{19}x + a_{20}y + a_{21}t + a_{22})) \\ &+ a_{23}\exp(a_{19}x + a_{20}y + a_{21}t + a_{22}), \end{aligned} \tag{2.3}$$

where

$$g = a_1 x + a_2 y + a_3 t + a_4, \quad h = a_5 x + a_6 y + a_7 t + a_8,$$

and the real numbers a_i, $i = 1, \ldots, 23$ will have to be determined later on.